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# MANUAL OF PHYSICAL MEASUREMENTS

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# A MANUAL

OF

# PHYSICAL MEASUREMENTS

BY

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*Professor of Physics in the University of Minnesota*

AND

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# PREFACE TO FOURTH EDITION

This manual is an outline of the laboratory experiments given in the courses in general Physics at the University of Minnesota. The laboratory work, in these courses, supplements the lectures and recitations.

The experiments given in a junior course of one quarter in electrical measurements are also included in the section on electricity.

It is taken for granted that the student has acquired a general knowledge of a subject before it is considered in the laboratory and no attempt is made in the manual at completeness in subject-matter or in explanations. The work is done under the guidance of an instructor who furnishes any additional information necessary.

The student should feel that acceptable results depend upon his own ability to properly adjust the apparatus, and he alone should plan and execute the details of the experiments, subject of course to the criticism of the instructor.

We here wish to thank Professors L. W. McKeehan, John T. Tate and L. F. Miller for valuable suggestions, criticisms, and assistance during the preparation of this manual.

ANTHONY ZELENY,
HENRY A. ERIKSON.

# INTRODUCTION

Work in the Physical Laboratory brings the student into first-hand touch with physical principles and physical apparatus, and the impressions produced through the senses furnish a solid foundation for further study.

The close attention to every detail and the exercise of deliberate judgment which are required in every experiment if a worthy result is to be obtained, tend to produce a habit of accuracy which is of inestimable value. This training is obtained only when every effort is made to get the very best result that the time allowed for the experiment and the apparatus employed will permit.

Before observations on any experiment are begun, the theory of the experiment should be mastered as well as the functions of the various parts of the apparatus which is to be used. Without such study, necessary observations may be omitted or taken in the wrong way, and apparatus whose value depends upon its accuracy may be injured permanently because of a lack of knowledge of its delicate parts. Furthermore it is important to determine the degree of accuracy of every result which is obtained.

**Errors.**—For various reasons it is impossible to obtain the absolute value of an unknown quantity. Any measurement is affected, to a greater or less extent, by errors which may be classified under the two heads: *constant* and *accidental*.

As examples of *constant* errors may be mentioned the following:

Physical errors, or errors which arise out of physical sources, e.g., change of length of a steel tape with tem-

perature; instrumental errors, such as faulty construction or adjustment of apparatus; personal errors, or the personal equation; blunders. Constant errors can often be eliminated or corrected for.

Under *accidental* errors are grouped those which remain after the constant errors have been taken into account. It is not possible to determine the magnitude of the accidental error. The best that can be done is to determine the probable limits within which the true result lies.

A number of rules and methods for finding these limits have been developed in the Theory of Least Squares.

The rigid application of these rules in the elementary laboratory is hardly justifiable. However some simple method for determining the first figure in the result affected by the accidental error is necessary, and, therefore, the mastery of the following is required.

**Error in a Single Reading.**—Instruments should be read to a fraction of their smallest division and hence the last figure in the reading is an estimated quantity, the accuracy of which depends upon the experience of the observer and the size of the smallest division. Estimate fractions in tenths, and record decimally. In general the estimation is liable to be correct to within one of the estimated parts. The quantity so estimated must be included in the result as the last *significant* figure and one of the estimated parts may be taken as the probable limit of error in the reading. For example, if in the reading 2.56 the last figure was obtained by estimating, the reading is liable to contain an error as large as 0.01. This fact may be expressed by writing the result $2.56 \pm 1$ meaning that the true value probably lies between $2.56 + .01$ and $2.56 - .01$.

**Error in an Average.**—If several readings of the same quantity are taken they will differ among themselves

because of the estimated part and also because of the inability to obtain each time the same conditions or the same adjustment of the apparatus.

The most probable value is the arithmetical mean obtained from several such readings, and the probable error as given by the theory of least squares, is

$$E = \pm 0.67 \sqrt{\frac{\Sigma v^2}{n\,(n-1)}},$$

where $n$ is the number of observations and $\Sigma v^2$ the sum of the squares of the differences between the arithmetical mean and each of the readings.

The above equation is, however, too laborious for general use in an elementary laboratory and therefore the following simpler though less accurate expression* for the probable error may be used:

$$E = \pm \frac{0.85}{n\sqrt{n-1}} \Sigma v = K \Sigma v,$$

where $\Sigma v$ is the *arithmetical* sum of the differences (regardless of sign).

When $n =$ 2, 3, 4, 5, 6, 7, 8, 9, 10,
$K =$ .425, .200, .123, .085, .063. .049, .040, .033, .028.

Suppose the following to be the results of a repeated measurement:

| *Length* | $v$ |
|---|---|
| 2.53 | .008 |
| 2.55 | .012 |
| 2.54 | .002 |
| 2.54 | .002 |
| 2.53 | .008 |
| The mean = 2.538 | .032 = $\Sigma v$. |

Since $n = 5$, $E = \pm .085 \times .032 = \pm .002$.
The result is written

$$2.538 \pm 2.$$

* Method of Least Squares.—Merriman.

The probable *numerical* error in this case is .002, and the probable *percentage* of error is 0.08%.

All figures to the right of the one affected by the error are of no value and should be omitted. Hence, if 10.5824 has an error of one in the figure 8 it should be written $10.58 \pm 1$, and if the number 10582465 has an error of 3 in the figure 8 it should be expressed as $10580000 \pm 30000$ or $(10.58 \pm 3) \times 10^6$.

**Error in Computed Results.—**

The effect of the observational error on a result computed from several observed quantities may be obtained approximately in the following manner:

In addition or subtraction retain in the sum or difference the largest *numerical* error in the quantities involved. Thus

$$\begin{array}{l} 45.23 \pm 8 \\ 25.6 \pm 2 \\ \underline{2.588 \pm 1} \\ 73.4 \pm 2 \end{array} \qquad \begin{array}{l} 45.23 \pm 8 \\ \underline{25.6 \pm 2} \\ 19.6 \pm 2 \end{array}$$

In multiplication or division retain in the product or quotient the *percentage* of error of the least accurate factor involved. Thus

$$(4.05 \pm 2) \times (60.41 \pm 3) = 245. \pm 1$$
$$(60.41 \pm 3) \div (4.05 \pm 2) = 14.91 \pm 7$$

The largest probable error in each case is in the factor 4.05 and amounts to about 0.5%. Taking 0.5% as the error gives in the product $\pm 1$ in the units place and in the quotient $\pm 7$ in the second decimal place.

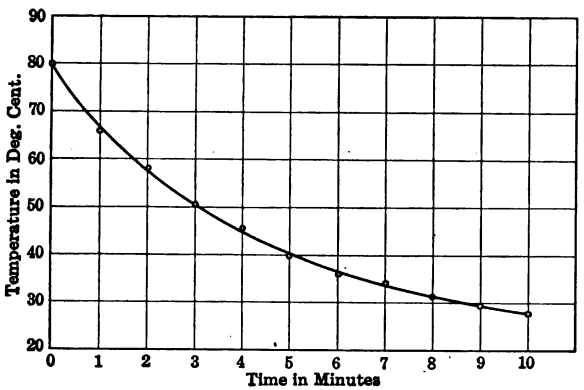
When raising a quantity to any power multiply the percentage of error in the quantity by the exponent. Thus

$$(a \pm e\%)^n = a^n \pm ne\%.$$

For example, if the number is squared, the percentage

error is doubled and if a square root is taken the percentage error is one-half that of the number.

**Graphical Representation of Results.**—The relation between two varying quantities may be more clearly shown by plotting them on coördinate paper, one quantity being represented by distances parallel to the $X$ axis and the other by distances parallel to the $Y$ axis. The choice of units to be used for laying off these quantities is arbitrary and need not be the same for the two axes. In order to obtain, however, the most complete representation the unit should be so chosen, where practicable, that the smallest division on the coördinate paper represents a unit of the last significant figure of the quantity. The unit selected should be recorded along each of the axes.




A scale marking convenient division-points, equally spaced, expressed in the units of the quantity to be plotted in that direction, should be placed along each axis, as shown in the figure.

The individual readings are not recorded on the coördinate paper but are tabulated on a separate sheet. The points are marked clearly by crosses or circles surrounding the points.

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# MECHANICS

## EXPERIMENT 1

### TO MEASURE A LENGTH IN CENTIMETERS AND IN INCHES DIRECTLY BY MEANS OF A SCALE

**Apparatus.**—A metric scale; an English scale; a cylinder whose length is to be measured.

**Method and Manipulation.**—Place the cylinder against the side of the scale so that the two are parallel, and so that the scale extends beyond the ends of the cylinder. Do not try to adjust until a line on the scale is even with an end of the cylinder. The distance that the cylinder extends into a division must be determined by estimating to tenths. A division is the space between the centers of two adjacent lines on the scale. Divide this distance mentally into halves and the half into which the cylinder or object extends into five parts, each of which then is a tenth of the whole division.

The difference between the readings obtained for th two ends, reckoned in each case from the zero end of the scale, gives the length of the cylinder.

In repeating observations place the cylinder each time opposite a different portion of the scale.

Obtain an average length in centimeters by taking several readings on the scale on which the smallest division is a millimeter and likewise on the scale on which the smallest division is a sixteenth of an inch. Solve for the number of centimeters in an inch and find the last significant figure in result by applying the rules given in the introductory chapter.

**Sources of Error.**—The scale and object may not be parallel.

The point read on the scale may not be exactly opposite the end of the object. This error is generally due to parallax, i.e., the eye is not in a line perpendicular to the scale at the point where the reading is taken.

## EXPERIMENT 2

### TO DETERMINE THE VOLUME OF A CYLINDER

**Method.**—The length and diameter of the cylinder are measured, and from these quantities the volume is computed.

**Apparatus.**—Vernier caliper; micrometer caliper with holder; magnifying glass; the cylinder used in Exp. 1.

I. TO MEASURE THE LENGTH OF THE CYLINDER WITH THE VERNIER CALIPER.

The *vernier caliper*, Fig. 1, consists of a graduated

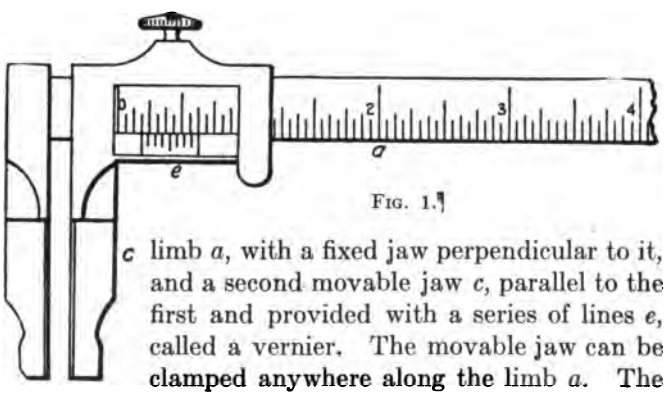



FIG. 1.

limb *a*, with a fixed jaw perpendicular to it, and a second movable jaw *c*, parallel to the first and provided with a series of lines *e*, called a vernier. The movable jaw can be clamped anywhere along the limb *a*. The clamping screw is rarely used and has been removed from the calipers supplied. Since the initial line on the vernier (at the left in the figure) coincides with the zero line on the scale of the limb when the jaws are together, the

position of this line at any time gives on the scale the separation of the jaws. When the initial line falls between two divisions on the scale, the fraction of a division is not estimated by the eye, but is measured by means of the vernier.

**The Vernier.**

Let $l$ = the length of a division on the scale,
$v$ = the length of a division on the vernier,
$n$ = the number of divisions on the vernier.

The number of divisions $n$, on the vernier equals in length $(n-1)$ divisions on the scale of the limb.

In Fig. 2 the initial line of the vernier coincides with line 4 on the scale. Line 1 is, then, the distance $(l-v)$ from line $a$; line 2 is the distance $2(l-v)$ from $b$; etc.

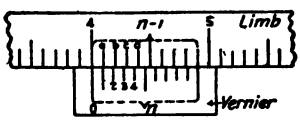



FIG. 2.

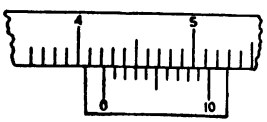



FIG. 3.

If now the vernier is moved until line 1 coincides with the line $a$, the initial line of the vernier is the distance $(l-v)$ from line 4. In this case the reading of the initial line is $4+(l-v)$. If line 2 coincides with $b$, the initial line is $2(l-v)$ from 4; etc. It is seen that the fraction of a division the initial line is away from $4=(l-v)$ multiplied by the number of the vernier line which exactly coincides with a line on the scale. The quantity $(l-v)$ is called the "least count" of the vernier. The value of $(l-v)$ is determined in the following manner:

$vn=l(n-1)=ln-l$ (by construction of the vernier).

$$ln-vn=l$$

$$(l-v)=\frac{l}{n}$$

The least count thus equals the length of a division on the scale divided by the number of divisions on the vernier. The reading of the vernier in Fig. 3 is 4.23.

[On some verniers $n$ divisions do not equal $(n-1)$ divisions on the scale but some other whole number. The value of the least count in these cases is determined by similar reasoning.]

When no line on the vernier exactly coincides with a line on the scale, i.e., when two adjacent lines of the vernier are within a division on the scale, a fraction of the least count must be estimated.

When the vernier lines are at equal distances from the scale lines, as shown in Fig. *a*, the fraction of the least count is 0.5. In Fig. *b*, when one line appears twice as far from a scale line as the other, the reading is 0.3 of the least count; in Fig. *c* it is 0.2.

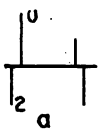


FIG. *a*.

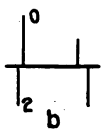


FIG. *b*.

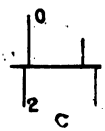


FIG. *c*.

**Manipulation.**—Find the least count of each vernier on the caliper. Bring the jaws together and note the reading of the initial line when the jaws touch.

The difference in the reading when the jaws are together and the reading when the object is between them, is the length of the object. Obtain the length of the cylinder in terms of each scale on the caliper. Reduce the length in inches (there will be as many of these as there are inch-reading verniers) to centimeters, and include them in the final average.

**Sources of Error.**—1. Jaws may not be parallel.

2. The object measured may not be parallel to the scale.

3. Too great pressure may bend jaws or change dimensions of the object measured.

4. Dust between the jaws of the calipers may cause the zero readings to be too great.

II. To MEASURE THE DIAMETER OF THE CYLINDER WITH THE MICROMETER CALIPER.

The *micrometer caliper*, Fig. 4, consists of a uniform screw moving in a fixed nut.

The nut *d*, has a scale *f*, and carries the arm *a*. The head of the screw *b*, with its divisions at *e*, is called the micrometer head. The end of the screw *c*, can be moved against any object placed in the opening *m*.

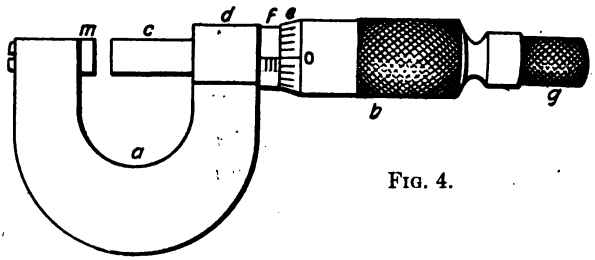



FIG. 4.

Examine the caliper and determine the following quantities:

1. Length of a division on the linear scale on the nut.
2. The pitch of the screw.
3. Number of divisions on the micrometer head.
4. The distance advanced by the screw when it is turned through one division on the micrometer head.
5. The fraction of a millimeter that can be obtained by estimating.

**Manipulation.**—To facilitate the taking of readings place the caliper in the holder. Determine the zero reading of the micrometer. Do not force the screw. Stop when the head first touches the object. A friction head *g*, if provided, insures the application of the proper

pressure. Turn until the friction head begins to move, or, if there is a ratchet attachment, until it clicks.

Hold the cylinder loosely between the jaws of the caliper. This practically insures getting the cylinder perpendicular to the jaws.

Measure the diameter at several points along the length of the cylinder and determine the average diameter.

**Sources of Error.**—1. The pitch of the screw may not be known accurately, and may vary in different parts.

2. Change of temperature of the caliper if held by hand may change the zero reading during the experiment.

III. COMPUTE THE VOLUME OF THE CYLINDER IN CUBIC CENTIMETERS FROM THE AVERAGE VALUES OF THE LENGTH AND DIAMETER OBTAINED ABOVE.

If L is the length and D is the diameter of a right circular cylinder, its volume V is given by the formula

$$V = \frac{\pi D^2 L}{4}$$

Determine the last significant figure in the result by applying the rules given in the introductory chapter.

## EXPERIMENT 3

### TO DETERMINE THE RADIUS OF CURVATURE OF A SPHERICAL SURFACE BY MEANS OF A SPHEROMETER

**Apparatus.**—A spherometer; a piece of plate glass; a sheet of paper; dividers; metric scale; a spherical surface.

The *spherometer* consists of a screw with a micrometer head. This screw turns in a nut which is supported by three legs at equal distances from each other. The screw and legs end in blunt points. The former is in the center of the triangle formed by the latter. A vertical

scale attached to the nut is used to determine the number of whole revolutions of the micrometer head.

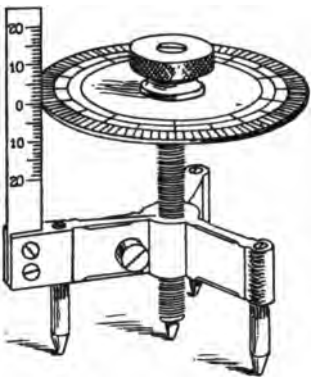



FIG. 5.

Determine the following quantities:

1. Length of a division on the vertical scale.
2. Pitch of the screw.
3. Value of a division on the micrometer head.

**Method and Manipulation.**—The spherometer is placed on the plate glass, and the micrometer head is turned until the screw just touches the plate, and is thus in the same plane as the tips of the legs. To determine this condition, press very lightly on the nut with two fingers, one over each of two legs, and tap with the other hand over the third leg. If the spherometer hobbles the screw is too low, and if it does not, it is likely to be too high. Find the point where the spherometer will not hobble, but will do so if the micrometer head is turned through a tenth of a division. The reading of the scale and micrometer at the former position gives the *zero reading*.

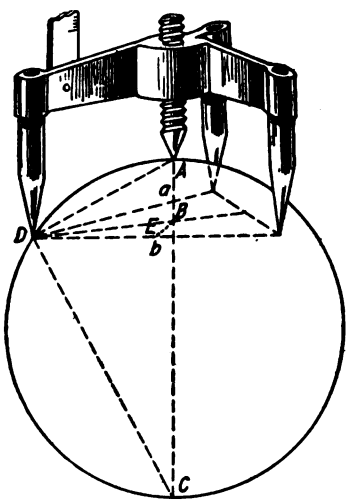



FIG. 6.

The spherometer is then placed on the spherical surface. The micrometer head is again turned until the four points touch the surface. The scale and micrometer are read, and the distance $a$ of Fig. 6, through which the screw moved from its zero position, is determined.

Turn the screw until the four points are again in the same plane.

Place the instrument on a piece of paper, and press lightly until the four points make small dents in the paper. By aid of dividers and scale, the average distance $DB$ between the legs and the point of the screw is determined.

The radius of curvature is calculated from the distances $a$ and $DB$.

**Suggestion.**—The vertical scale usually has the zero mark at the center. This is liable to confuse beginners. It is better to call the highest mark on the scale zero.

## EXPERIMENT 4

### TO DETERMINE THE MASS OF A BODY

**Apparatus.**—The analytical balance.

The arrangement of parts is as shown in the figure. The beam is usually supported by an agate knife-edge on an agate surface. The beam can be raised by means of a milled head in front of the case so that the knife-edge is not in contact with the supporting surface.

There are two adjustments that should be understood. The sensitiveness of the balance is dependent upon the distance between the knife-edge and the center of gravity of the beam. The shorter this distance is, the greater the sensitiveness. This distance can be changed by raising or lowering the nut that is on the vertical screw above the beam.

Attached to the beam is a horizontal screw with a movable nut. By the moving of this nut, the beam may be adjusted so that the pointer comes to rest at the center of the lower scale.

On the beam, or attached to it, is a scale on which the distance between the central knife-edge and the edge supporting the scale pan is usually divided into

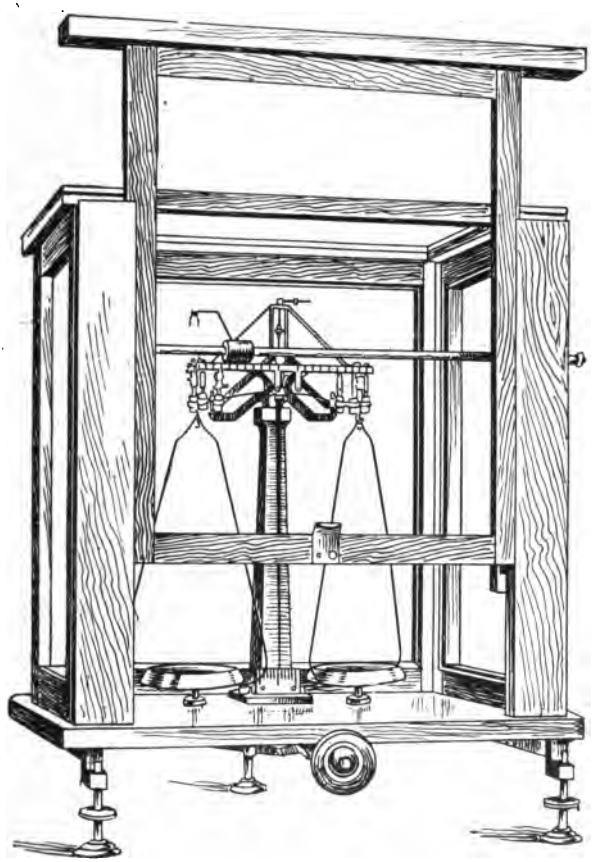


FIG. 7.

ten major parts. Smaller divisions, if present, are usually not used. Over this scale hangs a rider which can be placed at any point on the scale. This rider

weighs ten milligrams and is, therefore, when placed at the first division on the scale, equivalent to one milligram placed in the scale pan; when at the second, to two milligrams; etc. Brass weights are used for denominations ranging from the largest down to one gram. For fractions of a gram down to .01 gram or ten milligrams, platinum or aluminum weights are used. Less than ten milligrams is obtained by means of the rider.

**Method and Manipulation.**—Release the beam carefully, and if it is at rest, set it in vibration by raising and again releasing. Note at what place the pointer turns, counting the divisions from the left end of the scale and estimating to tenths of a division. Take five successive turning points. Call $a$, $c$, $e$, the readings on one side, and $b$, $d$, the readings on the other. Then the reading on the scale at which the pointer would stop is

$$Z = \frac{\frac{b+d}{2} + \frac{a+c+e}{3}}{2}.$$

This is called the *zero point.* It is necessary to take one more reading on one side because of the decrease in the vibrations due to friction.

Raise the beam, place the body to be weighed in the *left* pan, and add weights to the right until the pointer swings within the scale. Add weights only when the beam is raised, and try the effect of each addition by very carefully lowering the beam and noting which way the pointer begins to move. Take readings for turning points and average as in the case above. This gives the *point of rest* ($R$). If the point of rest happens to be the same as the zero point, the mass of the weights is equivalent to that of the body. If, however, the point of rest is less than the zero point, the mass of the weights

is too large, and if the point of rest is larger, the mass of the weights added is too small. In the last two cases we must determine what weight would have to be subtracted or added in order to bring the pointer to the zero reading. In order to do this we must know what is called the *sensibility* of the balance, i.e., the deflection of the pointer that one milligram will produce. To obtain this, change the weights by one milligram, by means of the rider, and determine, in the same manner as above, another point of rest which we will call the *sensibility point* (*S*). The difference between the *point of rest* and *sensibility point* gives the *sensibility*.

The weight to be added or subtracted is numerically equal to the difference between the zero point and the point of rest, divided by the sensibility.

**Suggestions.**—In getting the sensibility, add one milligram if the point of rest is larger than the zero point, and subtract if it is less.

Always use the largest weights permissible. This facilitates the weighing, as it reduces the number of separate weights needed and reserves the smaller weights for the final adjustment.

The method described above is usually spoken of as the *method of vibrations*.

**Errors and Precautions.**—Before changing the weight, and when through weighing, always raise the beam of the balance. This should be done when the pointer is at the center of the scale.

Since air currents influence the swing of the beam, readings must not be taken immediately after closing the case of the balance.

Take the reading for the turning point when the eye, the pointer and its image in the mirror back of the scale, are in the same line. This avoids the error due to parallax.

Always handle the weights with the forceps. *Never* touch them with the fingers.

When through with a weight, *always* return it to the proper place in the case.

As the zero point is affected by temperature changes and slight shifting of the parts of the balance, a zero point must be determined after the load has been removed. The average of the first and last zeros is used in the interpolation.

## EXPERIMENT 5

### RATIO OF THE LENGTHS OF THE BALANCE ARMS

**Method.**—Place a body having a mass of about fifty grams in the left pan, and obtain the apparent mass, using the method of vibrations. Then obtain the apparent mass when the body is in the right pan.

If $w$ = the mass of the body,
$a$ = the apparent mass of the body when in the left pan,
$b$ = the apparent mass of the body when in the right pan,
$r$ = the length of the right arm,
$l$ = the length of the left arm,

then $$wl = ar$$

and $$wr = bl$$

$$\therefore \frac{r}{l} = \sqrt{\frac{b}{a}}$$

**Remarks.**—If the left pan is used for the body, the apparent mass must be multiplied by the ratio $\frac{r}{l}$ in order to obtain the correct mass, since $w = a\frac{r}{l}$.

If the ratio is not known, the correct mass can be obtained by resorting to double weighing as above and

taking the square root of the product of the apparent masses, i.e., $w = \sqrt{ab}$.

**To Obtain the Weight in Vacuo.**—In air the body and the weights are lighter by the weight of the air displaced by each.

If $M$ = the weight of the body in vacuo,
$w$ = the apparent weight of the body in air,
$\lambda$ = the density of the air, See Table 19,
$\delta$ = the density of the weights,
$S$ = the density of the body,

then

$$M = w\left(1 + \frac{\lambda}{S} - \frac{\lambda}{\delta}\right)$$

## EXPERIMENT 6

### A STUDY OF ACCELERATED MOTION

**Apparatus.**—An inclined metal track about ten feet long, between the rails of which is placed an insulated metal strip, of length equal to the track, and over which is a paper ribbon; a car, mounted on ball bearings, from which a metal point projects toward the paper ribbon; an induction coil the primary of which is closed every second by means of a relay in the clock circuit, the secondary being connected to the track and to the metal strip. Closing the primary releases the car by attracting the armature of an electromagnet and causes an electric discharge to take place each second thereafter between the point projecting from the car, and the metal strip beneath it, thus puncturing the paper ribbon and marking the position of the car, at successive seconds.

**Method and Manipulation.**—Attach the car at the upper end of the track and close the primary circuit. As the car descends, the punctures through the paper

ribbon mark distances traveled by the car during each consecutive second.

By means of a meter scale measure the distance traversed.

Plot the total distances as abscissæ and the first, second and third powers of the corresponding times as ordinates. A straight line means that the distances ($S$) are proportional to ($t^x$) the corresponding ordinates.



FIG. 8.

From $s = \frac{1}{2} at^2$ compute the acceleration ($a$) for each time $t$ and average.

Obtain the difference between the consecutive distances and average. As the difference between two consecutive distances equals the acceleration, this average should check with the preceding average.

From the inclination of the plane and the acceleration compute the acceleration of gravity.

## EXPERIMENT 7

### EQUILIBRIUM OF PARALLEL FORCES

**Apparatus.**—A graduated bar (conveniently a meter stick) is suspended by two spiral springs whose supports are adjustable by means of turn-buckles; level; weight

trays and a number of known masses each of the order of a kilogram.

**Method and Manipulation.**—Determine the mass of the bar and consider this mass as concentrated at the center of the bar.

Suspend and measure the length of the two springs while hanging empty. This is done by placing the points of a divider in small holes, one at the upper end and one

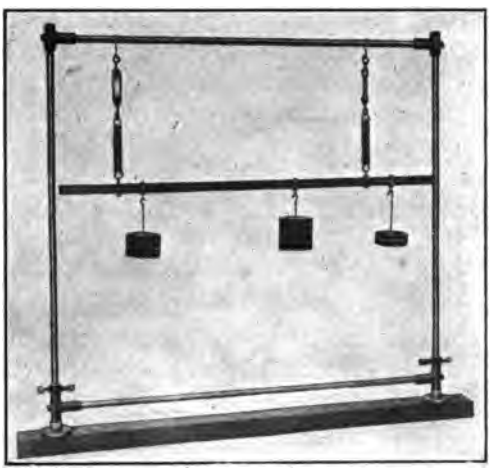


FIG. 9.

at the lower end of the spiral. The distance between the points is obtained from a steel scale. Suspend the bar by means of the two springs and stirrups placed a short distance from the ends of the bar. Suspend a tray carrying not less than one kilogram at each of three points between or outside the points of attachment of the springs.

By means of the turn-buckles adjust until the bar is level. Determine again the length of each spring, also

read on the bar the point of attachment of the springs and trays.

**Calibration of Springs.**—In order to determine the force pulling on each of the springs it is necessary to determine the relation between the elongation of the spring and the force applied. To do this suspend the spring and after measuring its length empty apply successively masses from one to four kilograms and measure the length of the extended spring in each case. According to Hooke's Law the elongation of the spring should be proportional to the forces applied. Having obtained the proper data, plot a curve for each spring, using the masses as abscissæ and the corresponding elongations as ordinates.

These calibration curves will now be used to determine the forces acting when the springs were attached to the bar.

*First Condition of Equilibrium* $\Sigma\ (F \cos \theta) = 0$.—Add algebraically all the forces, expressed in grams weight, acting upon the bar, those acting upward being taken as positive. Determine the component of each of the forces in any other given direction and obtain the algebraic sum.

The above sums by the first condition of equilibrium should equal zero. Any remainder obtained is a measure of the experimental error.

*Second Condition of Equilibrium* $\Sigma\ (Fr \cos \theta) = 0$.—From the readings for the points of application of the forces obtain the distance ($r$) of each force from any point on the bar. Obtain the algebraic sum of all the moments calling the counter clockwise moments positive.

By the second law of equilibrium this sum should equal zero. Note that this is true for any point on or off the bar.

## EXPERIMENT 8

### EQUILIBRIUM OF CONCURRENT FORCES

To test the condition for the equilibrium of forces which act at a common point.

**Apparatus.**—A force-table with four pulleys adjustable about a horizontal graduated circle upon which their angular positions may be read; a connecting ring with centering-pin; cords, weight-pans, and known masses.



FIG. 10.

**Method and Manipulation.**—Level the force-table approximately; i.e., by eye. A more accurate method of leveling is given on page 218, line 33, but an approximate method is here sufficient.

Connect three known masses to the ring, temporarily held at the center of the table by the centering-pin. Select unequal masses of not less than 200 grams, including the weight-pan. Find by trial position of the

pulleys for which the ring will return to the center if tapped vertically after any small displacement. Record the masses and corresponding angular readings. Change the position of one pulley by any amount (after re-inserting the centering-pin, if this has been removed) and readjust by moving the other pulleys. Record the new positions. Repeat several times in this way and average the corresponding angles obtained.

Connect four known masses to the connecting ring and proceed as above, except that it is useless to repeat readings as there directed, since a quadrilateral force-polygon is not completely determined by the magnitude of the four forces composing it, whereas a triangular force-polygon is so determined.

**Calculations.**—Obtain the average angle between the adjacent forces in each case tested. Then proceed as follows:

Construct the force-polygon for each case and from it find the resultant of the forces. This is theoretically equal to zero, so the amount of the resultant is the experimental error.

Compute the rectangular components referred to the same pair of axes, of all the forces in each case. Add the corresponding components with proper regard to sign, and from the sums compute the amount and direction of the resultant.

**Note on Graphical Computations.**—Use a well-sharpened pencil (4H or 6H) and put no ink-lines over the pencil-lines. Letter diagrams in ink, however, and put small circles, in ink, about points plotted from data or referred to in the discussion of the graph.

## EXPERIMENT 9

### EQUILIBRIUM OF NON-CONCURRENT FORCES—LADDER

**Purpose.**—To test the conditions for the equilibrium of forces which do not act in directions passing through the same point.

**Apparatus.**—A model ladder supported in a vertical plane by horizontal wires at each end and by a vertical wire at the lower end; a weight to be hung on any

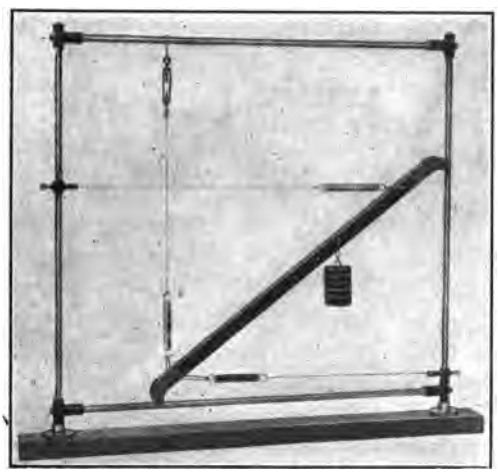


FIG. 11.

"rung" of the ladder; spring dynamometers by which the force along each wire may be measured; a large pair of calipers, or beam compasses, for obtaining the rectangular coördinates of the points at which forces act; a pair of dividers.

**Method and Manipulations.**—The ladder, with the weight hung upon it, is placed at any convenient angle to the horizontal, and the points at which the supporting

wires are attached to the frame are adjusted until these wires are all either vertical or horizontal, i.e., parallel to the frame. The coördinates of the points at which forces are applied to the ladder (including coördinates of the center of mass of the unloaded ladder, which is marked by a brass-headed nail) are measured by the caliper, the left-hand and lower bars of the frame forming convenient axes of reference. The length of each spring dynamometer is measured three times by means of the divider and scale, as follows:

$A$ = the length when stretched by the force $F$ applied to the ladder,

$B$ = the length when stretched by a known force $f$ (a set of weights hung on a weight-pan),

$C$ = the length when unstretched.

Then, assuming that the extension is proportional to the force producing it (Hooke's Law), we have the following equation for $F$:

$$F = f\frac{A - C}{B - C}.$$

An alternative method is to calibrate each spring as directed in Exp. 7 and to determine the force $F$ from the calibration curve. The weight of the cylinder hung upon the ladder, as well as the weight of the ladder itself, must be obtained.

**Calculations.**—Plot on coördinate paper the positions of the points of application of all the forces acting on the ladder. Compute the amounts of the forces in the three wires.

*First Condition of Equilibrium.*—Find the sum, with proper regard to sign, of all the horizontal, and of all the vertical forces.

*Second Condition of Equilibrium.*—Compute the total torque about each end of the ladder. All these results

should be small if the measurements have been accurately obtained.

## EXPERIMENT 10

### COEFFICIENT OF SLIDING FRICTION

**Apparatus.**—An inclined plane having means for clamping it at any angle to the horizontal, and provided with an adjustable pulley at the upper end; a heavy block of known mass, with cord, weight-pan, and known weights for applying forces to the block in a direction parallel to the plane; a steel scale for measuring the sides



FIG. 12.

of the triangle formed by three points marked on the edges of the inclined plane and its horizontal base; several flat sheets of different materials.

**Method and Manipulation.**—Place the surface under investigation upon the inclined plane, and place the block upon it. Connect a known mass to the cord passing over the pulley and adjust the inclination of the

plane until the block will just continue to slide up the plane (without acceleration) when given a slight push in that direction. Measure the sides of the triangle formed by the marked points on the edges of the inclined plane and its base. Readjust using the same known mass if possible until the block will just continue to slide down the plane (without acceleration) when given a slight push in that direction. Measure the sides of the new triangle now formed by the marked points. Care should be taken to elevate or depress both sides of the plane by the same amount in making an adjustment, and that the cord passing over the pulley is parallel to the plane. Repeat with each of the different pairs of surfaces available.

**Calculations.**—Construct a triangle with the three measured lengths found in each case tested, using as large a scale as possible, and placing the side corresponding to the base of the plane in a horizontal position on the paper. Construct the force-polygon for each case as follows:

1. Draw a vertical line from any point on the side of the triangle which corresponds to the inclined plane, and lay off upon it to a convenient scale the weight of the block (which will be given).

2. Lay off up the side of the triangle from the base of the first line the weight hung over the pulley.

3. From the upper end of the first line draw a line parallel to the plane.

4. From the upper end of the second line draw a perpendicular to the plane, and intersecting the third line.

The polygon will now be complete; indicate the directions of the four forces by arrowheads, and find the amounts of the force of friction and of the normal pressure between the block and the plane by measuring the segments of the third and fourth lines of the polygon as

drawn above; the coefficient of sliding friction is the ratio of the former to the latter.

Find the coefficient of sliding friction between various pairs of surfaces.

## EXPERIMENT 11

### LOCATION OF THE CENTER OF MASS OF A WHEEL

**Apparatus.**—A wheel and axle; a leveling way or track along which the axle may roll freely; a double plumb-line and weight of the order of 5 grams.

**Method and Manipulation.**—(*a*) Level the track upon which the wheel rests.

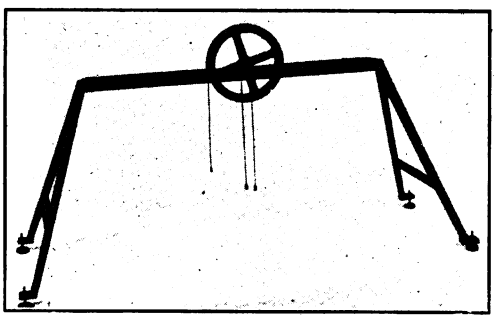


FIG. 13.

(*b*) Hang the double plumb-line over the shaft, let the wheel vibrate until it comes to rest, and carefully mark the points $aa'$ at which the plumb-lines cross the rim of the wheel.

(*c*) Hang a known weight ($w$) from the rim of the wheel by means of a string passing over the circumference; a small amount of soft wax will serve for attaching the string. Again let the wheel vibrate and come to rest and mark the points $bb'$ at which the plumb-lines cross the rim. Measure the arcs $ab$ and $a'b'$.

(*d*) Repeat (*c*) with the weight suspended on the opposite side of the wheel so as to eliminate to some extent errors due to inaccurate leveling of the track.

(*e*) Determine (*w*) the mass of the wheel.

(*f*) Measure the radius (*R*) of the wheel.

(*g*) With a flexible steel tape measure the arcs *ab* and *a'b'* and average these with the corresponding arcs obtained from the reverse suspension.

**Computations and Results.**—Let *A* be the point midway between the two points *aa'*, and *B* the point midway between the points *bb.'* A line drawn from *O*, the center of the wheel, to *A* passes through the center of mass of the wheel, and the distance *x* of the center of mass from *O* is determined by the equation

$$wR = Wx \sin \overline{AOB}$$

in which *w*, *W*, and *R* are known, and the angle $\overline{AOB}$ is determined from the average of the arcs *ab* and *a'b'*. Compute *x* and the mass which must be removed from the rim of the wheel in order to cause it to balance.

## EXPERIMENT 12

### TO TEST HOOKE'S LAW AND TO DETERMINE YOUNG'S MODULUS IN THE CASE OF THE BENDING OF A ROD

**Apparatus.**—A rod supported by a fulcrum at each end and provided with a weight-pan at the center; a stage micrometer and microscope arranged for measuring vertical distances; a number of weights.

**Method and Manipulation.**—Adjust the rod so that it lies firmly on the fulcrums. Focus the microscope and turn the micrometer until the cross-hair is on the fine reference point on the rod, being careful to approach the point so as to avoid the error due to back-lash of

the micrometer screw. Obtain the micrometer reading $R_o$, the weight-pan being empty. Add a known mass to the pan, adjust and again read the micrometer. Continue thus until six or eight known masses have been added. Repeat the above observations by again beginning with no load in the weight-pan. Care must be exercised so as not to jar the apparatus in the meantime.

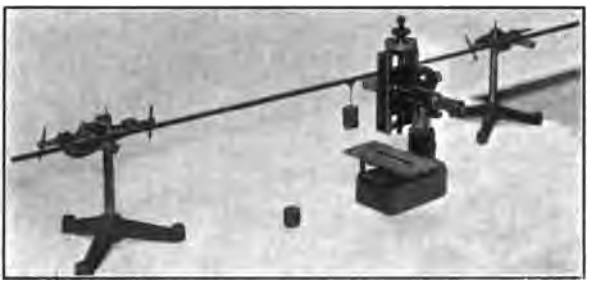


FIG. 14.

**Graphical Interpretation of Observations.**—The relation between the loads and deflections may best be shown by the use of coördinate paper. Plot the loads as abscissæ and the one-half, the first, and the second powers of the deflections as ordinates. (See Graphical Representation of Results in introductory chapter.)

If the curve is a straight line in the case of the first power the relation is that the deflections vary directly as the loads, the quantities being represented by homologous sides of similar triangles. This proportionality is expressed by Hooke's law, i.e., the strain is proportional to the stress.

**Young's Modulus.**—In the case of a rectangular bar of depth $d$, breadth $b$, and length $l$, supported at both ends Young's modulus

$$M = \frac{Fl^3}{4bd^3y}$$

where $F$ is the force in dynes applied at the middle and $y$ is the deflection at the middle. Measure the dimensions of the bar and compute $M$ for each of three values of $F$ and $y$ determined from the graph.

## EXPERIMENT 13

### ELASTICITY

To test Hooke's Law and to determine Young's modulus in the case of the stretching of an iron wire.



FIG. 15.

**Apparatus.**—The wire is attached to the ceiling and hangs vertically. It has a pan for weights attached at the lower end. A cross bar, clamped to the wire near the upper end, supports two vertical rods which in turn support a cross bar near the lower end of the wire and through which the wire passes freely. This bar supports, near one end, a vertical screw provided with a micrometer head, the head being beneath the bar. Above the bar a circular disc is clamped to the wire. A metal plate, provided with a level on top and two pointed feet beneath one end, is placed so that the feet rest on the circular disc and the other end rests on the upper end of the micrometer screw.

**Method and Manipulation.**—(*a*) To test Hooke's Law, i.e., that stress is proportional to the strain.

Place a mass of one kilogram in the weight-pan and

adjust the micrometer until the level reads zero. Read the micrometer. This gives the zero reading ($R_0$.), the wire being straight. Add an additional kilogram and again level and obtain the micrometer reading ($R$). $R - R_0$. gives the elongation or strain for the stress represented by the second weight. Continue in this manner until deviation from Hooke's Law begins, i.e., until the elastic limit has been reached. Prepare a graph of the above results by laying off the elongations as ordinates and the corresponding loads as abscissae. The straight portion of the graph represents the region over which Hooke's Law holds.

(*b*) To determine Young's modulus and the elastic limit.

Young's modulus is the stress per unit cross-sectional area divided by the strain per unit length of the wire, i.e.,

$$M = \frac{PL}{ae}$$

where $P$ represents the total load in dynes or poundals; $e$ the total elongation produced by the load $P$; $L$ the length and $a$ the cross-sectional area of the wire measured in terms of the centimeter or the inch.

Compute Young's modulus for a given load by obtaining the elongation from the graph.

Compute the load per unit area at the elastic limit.

## EXPERIMENT 14

### ELONGATION OF A SPIRAL SPRING

**Apparatus.**—A spiral spring, carrying a pan for weights, is suspended in front of a mirrored scale.

**Method.**—Readings are taken on the mirrored scale opposite the point of the lower hook when no weights are in the pan, and after each additional weight until

about four have been added. Each reading must be taken when the eye is in line with the point of the hook and its image in the mirror.

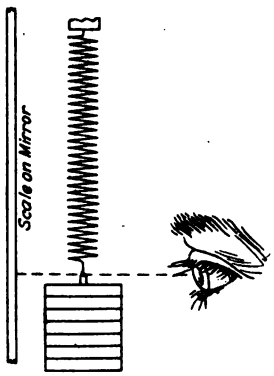



FIG. 16.

Plot the readings obtained, and determine if Hooke's law holds within the limits of observational error.

## EXPERIMENT 15

### HOOKE'S LAW AND SIMPLE HARMONIC MOTION

**Apparatus.**—Same as in Exp. 14, and a stop-watch.

**Method and Manipulation.**—In the preceding experiment it was found that

$$\frac{\text{Stress}}{\text{Strain}} = K, \text{a constant.}$$

When a spring carrying a constant load is elongated beyond the point of rest, the upward contracting force is greater than the force of gravity acting downward upon the attached mass. If the spring is shortened, the force of gravity is greater than the contracting force. At the point of rest the two forces are equal.

For any displacement $x$ of the spring from the point of rest, there is a force tending to restore it that is equal to $ma$, where $a$ is the acceleration it gives to the attached mass $m$.

From the last experiment

$$-\frac{ma}{x} = K, \text{ or } a = -K_1 x.$$

This is the condition for simple harmonic motion for which the general equation is

$$t = 2\pi\sqrt{-\frac{x}{a}}.$$

If the mass $m$ produces a static elongation $s$, then when the mass $m$ is vibrating vertically and is at a displacement up equal to $s$, the acceleration downward is $g$; therefore, at this instant

$$-\frac{x}{a} = \frac{s}{g}.$$

The equation for the time thus becomes

$$t = 2\pi\sqrt{\frac{s}{g}}.$$

Determine from the curve obtained in Exp. 14 the elongation $s$ that a mass $m$ will produce.

Substitute the value of $s$ in the equation, and compute the time $t$. Using the mass $m$ that produced the elongation $s$, cause it to vibrate vertically, and determine the time of vibration. This time should agree within the limits of observational error with the computed result.

## EXPERIMENT 16

### TORSION OF A ROD

**Apparatus.**—A rod is firmly attached at one end to the center of an iron cylinder and at the other to a graduated disc on which the angle of torsion may be read. At-

tached to the circumference of the disc is a cord to the free end of which a weight pan is connected.

**Method and Manipulation.**—Take the reading on the disc when the pan is empty, and after each additional

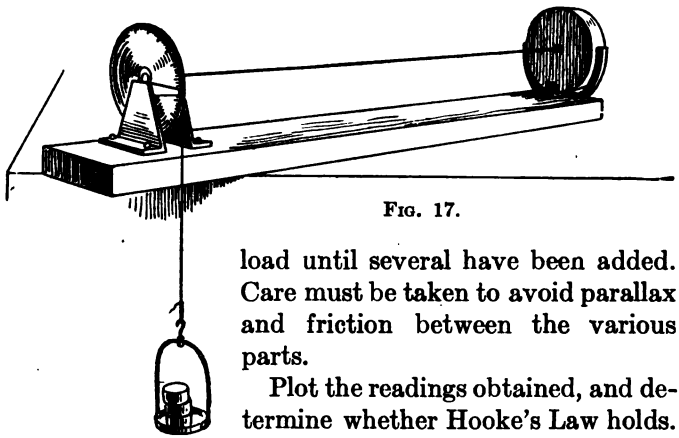


FIG. 17.

load until several have been added. Care must be taken to avoid parallax and friction between the various parts.

Plot the readings obtained, and determine whether Hooke's Law holds.

## EXPERIMENT 17

### HOOKE'S LAW AND SIMPLE HARMONIC MOTION

**Apparatus.**—Same as in Exp. 16, and in addition a stop-watch.

**Method.**—Since, as found in the preceding experiment, the strain is proportional to stress, the rod, when set in motion as a torsional pendulum, will vibrate with simple harmonic motion. In order to compute the time of vibration the general equation for the period of a simple harmonic motion,

$$t = 2\pi\sqrt{-\frac{x}{a}},$$

must be altered so as to involve quantities that can in this case be readily measured.

For an angular displacement $\theta$, a point at a distance $r$ from the axis has a linear displacement $x = r\theta$ Likewise if $\alpha$ is the angular acceleration, the linear acceleration at a distance $r$ is $a = r\alpha$,

$$\therefore \quad \frac{x}{a} = \frac{r\theta}{r\alpha} = \frac{\theta}{\alpha},$$

$$\therefore \quad t = 2\pi\sqrt{-\frac{\theta}{\alpha}}.$$

The moment of the couple necessary to produce a displacement $\theta$ is $T\theta$, where $T$ is the moment of torsion, i.e., the moment of the couple necessary to produce a displacement of a raidan, but

(Duff, p. 84.)

$$-T\theta = \alpha I$$

where $I$ is the moment of inertia of the system.

$$\therefore \quad \frac{x}{a} = \frac{\theta}{\alpha} = -\frac{I}{T},$$

and

$$t = 2\pi\sqrt{\frac{I}{T}}.$$

If, therefore, $I$ and $T$ are known, the period $t$ can be determined.

To obtain $T$ determine from the curve in Exp. 16 the number $n$ of circumference divisions in the angle of torsion produced by a mass $m$. This angle $\theta$, in radians, equals $\frac{n}{N}2\pi$, where $N$ is the number of divisions in the whole circumference. If $r$ is the radius of the graduated disc, then

$$T\theta = mgr,$$

$$\therefore \quad T = \frac{mgr}{\theta}.$$

Compute $I$ from the dimensions of the cylinder.

Substitute the values of $T$ and $I$ in equation and solve for $t$.

Remove the rod with its attached disc and cylinder, and suspend from bracket so as to form a torsion pendulum as shown in Fig. 18. Determine the time of vibration. This should check with the computed time if the vibration is simple harmonic.



FIG. 18.

From the equation

$$\mu = \frac{128\pi lI}{t^2 d^4}$$

compute the coefficient of simple rigidity of the material of the wire.

In the above equation $l$ is the length of the wire and $d$ its diameter.

## EXPERIMENT 18

### MOMENT OF INERTA OF AN IRREGULAR BODY ABOUT AN AXIS THROUGH ITS CENTER OF INERTIA

**Method.**—When the body is suspended by a wire whose moment of torsion is $T$, its time of vibration as a torsional pendulum is

$$t = 2\pi\sqrt{\frac{I}{T}}. \quad \text{(See Exp. 17.)}$$

If a body of known moment of inertia $I'$ is suspended from the same wire the time of vibration

$$t' = 2\pi\sqrt{\frac{I'}{T}},$$

$$\therefore \quad I = \frac{t^2}{t'^2} I'.$$

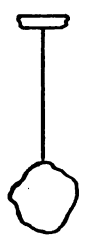


FIG. 19.

**Apparatus.**—The body whose moment of inertia is to be determined; a body whose moment can be calculated; a steel wire with proper attachments.

**Manipulation.**—Determine the time of vibration for each of the bodies to an accuracy comparable with the accuracy in the other measurements. Compute $I'$ from the mass and dimensions of the body. Solve for $I$.

## EXPERIMENT 19

### TO TEST EXPERIMENTALLY THE EQUATION $I = I_0 + Md^2$

**Method.**—The moment of inertia $I$ of a body may be determined experimentally by adding it to a torsional pendulum of known moment of inertia $I'$, and known time of vibration $t'$, and obtaining the new time of vibration $t$.

$$\text{Thus } t' = 2\pi\sqrt{\frac{I'}{T}}, \quad t = 2\pi\sqrt{\frac{I + I'}{T}},$$

$$\therefore \quad I = \frac{t^2 - t'^2}{t'^2} I'.$$

If two similar bodies of such form that their moments of inertia can be computed from their masses and dimensions be placed symmetrically on opposite sides of the torsional pendulum so as not to disturb its position, the equation $I = I_0 + Md^2$ can be tested experimentally.

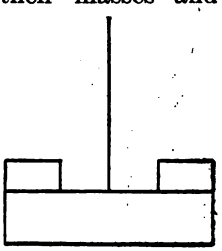


FIG. 20.

**Apparatus.**—The torsion pendulum used in Exp. 17; two small cylinders.

**Manipulation.**—Obtain by computation from its dimensions the moment of inertia $I'$ of the torsional pendulum, and determine its time of vibration.

Place the two cylinders on opposite sides of the pendulum so that their centers are equidistant from and in line with the wire, and obtain the time of vibration with the same degree of accuracy as above.

By means of the values $I'$, $t$ and $t'$, thus obtained, compute $I$, the moment of inertia of the cylinders about the axis around which they rotated.

Obtain the mass and dimensions of the cylinders and their distance $d$ from the axis of rotation. The value of $I$ obtained by substituting these values in the equation $I = I_0 + Md^2$ should agree to within the limits of observational error with the experimental result.

REFERENCE.—Duff, p. 66.

## EXPERIMENT 20

### ANGULAR ACCELERATION

**Apparatus.**—A heavy disc is mounted so that it may be rotated about its axis. A tuning fork is placed so that its vibrations may be traced by means of a stylus on the smoked face of the disc. The other face of the disc has a circle that is graduated to degrees. On each side of the disc is a magnifying glass fitted with a cross-hair. By the aid of these, the length of a given number of wave lengths on the tracing may be determined in degrees on the graduated circle.

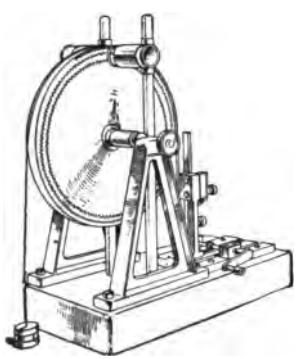


FIG. 21.

**Method and Manipulation.**—By means of a small amount of wax attach the string, and wind it on the disc. To the free end of the string, attach a weight of

about 100 grams. Adjust the disc so that the stylus will trace the vibrations of the fork on the smoked surface. Let the weight start the cylinder from rest, and then move the fork slowly so that the tracings will not overlap. As the velocity of the disc increases, the traced vibrations become longer. Let $\theta$, $\theta'$, $\theta''$, etc., equal consecutive angular lengths of forty vibrations. Let $n$ be the frequency of the fork. The lengths $\theta$, $\theta'$, $\theta''$, etc., are the consecutive angular distances the disc traveled during consecutive intervals of forty vibrations or $\frac{40}{n}$ sec. The difference $k$ between them is, therefore, the angular acceleration per $\frac{40}{n}$ sec. per $\frac{40}{n}$ sec.

Obtain an average value for $k$ from several measurements, and compute $\alpha$, the angular acceleration per second per second.

## EXPERIMENT 21

### TO TEST EXPERIMENTALLY THE RELATION $Fr = \alpha I$

**Apparatus.**—Same as in Exp. 20.

**Method and Manipulation.**—Obtain $\alpha$ the positive angular acceleration of the disc in the manner explained in Exp. 20.

In like manner obtain $\alpha_1$, the negative acceleration, i.e., the acceleration when the speed of the disc was decreasing because of friction.

Since the weight $M$ descends with an acceleration $a$, it acts on the cylinder with a force

$$P = M(g - a),$$

or since $a = \alpha r$

$$P = M(g - \alpha r)$$

where $g$ is the acceleration of gravity and $r$ is the radius

of the disc on which the string supporting the weight is wound.

Of this force an amount equal to $M\alpha_1 r$ is required to overcome the frictional resistance, and the remainder is effective in producing motion. Therefore the force required in the equation to be tested is

$$F = M[g - (\alpha + \alpha_1)r].$$

This $F$ times the radius of the drum gives the first member of the equation.

The value of the moment of inertia $I$ is computed from the expression for the moment of inertia of a cylinder about its axis.

$I$ multiplied by $\alpha$, gives the value of the second member which should check with the first.

Instead of correcting for friction by determining the negative acceleration, one may offset the resistance due to friction by adding a weight to the string that is just sufficient to cause the disc to move before the main weight is added. In this case $\alpha_1$, above becomes zero.

## EXPERIMENT 22

### ANGULAR VELOCITY

**Apparatus.**—Same as in Exp. 20.

**Method and Manipulation.**—Wind the string about the disc, and note the height of the weight. Release the disc, and the instant the weight strikes the floor let the stylus trace the vibrations of the fork on the smoked surface. Repeat several times. The length of one of the traced vibrations is the angular distance a particle on the disc traveled during one period of the fork,

If $\theta$ = the length of vibration in degrees,
$n$ = the frequency of the fork,
$\omega_1$ = the angular velocity in degrees per second,
then $\omega_1 = n\theta$.

In getting a value for $\theta$, measure the length of several vibrations and divide by the number.

Since on account of friction the velocity of the disc is not uniform, care must be taken to measure the vibrations, in each case, corresponding to the instant the weight strikes the floor.

From $\omega_1$ determine $\omega$ the angular velocity in radians per second.

## EXPERIMENT 23

### TO SHOW THAT $Fs = \frac{1}{2}\omega^2 I$

**Apparatus.**—Same as in Exp. 20.

**Method and Manipulation.**—The object is to show that the force $F$ producing rotation, from rest, in a body multiplied by the distance $s$ through which it acts is equal to the kinetic energy possessed by the body when the force ceases to act, i.e.,

$$Fs = \tfrac{1}{2}\omega^2 I,$$

where $\omega$ = the angular velocity,
$I$ = the moment of inertia of the body.

$F$ is the effective force, and must be found as in Exp. 21.

Obtain $\omega$ as in Exp. 22, after having offset friction by adding a supplementary weight as directed in Exp. 21, and in addition measure the distance $s$. Compute $I$ from the mass and dimensions of the disc. Substitute in the equation, and determine whether the equality holds within the limits of observational error.

## EXPERIMENT 24

### TO PROVE THAT THE MOMENTUM IS THE SAME AFTER AS IT IS BEFORE AN INELASTIC IMPACT

**Apparatus.**—Two metal cylinders are suspended from ceiling by parallel wires as shown in Fig. 22. One of

the cylinders is provided with fine points that during collision penetrate the lead surface of the other, thereby preventing the two from separating.

**Method and Manipulation.**—Adjust the cylinders until they are horizontal and in line with each other. Place the table so that when the cylinders are swinging the vertical rod beneath $B$ moves parallel and close to the wire $cd$. The wire $cd$ is leveled by adjusting so that it is even with the surfaces of the mercury contained in two communicating glass tubes.

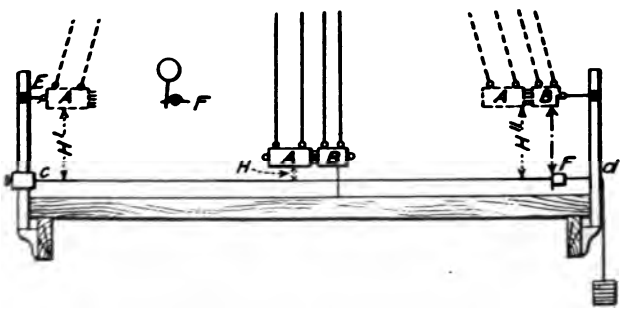



FIG. 22.

By means of a vertical mirrored scale obtain the height $H$. Raise cylinder $A$ and attach it at $E$. Obtain the height $H'$. Release $A$. After collision the two cylinders move on together, and the vertical rod beneath $B$ will leave at a definite point the paper index $f$ placed on the wire $cd$. By means of a thread draw the two bodies over until the vertical rod just touches the paper index and obtain the heights $H''$.

The horizontal velocity of $A$ just before impact is $v = \sqrt{2g(H' - H)}$ and the velocity of $A$ and $B$ after impact is $v' = \sqrt{2g\,(H'' - H)}$.

If the momentum is the same before and after impact,

$$m\sqrt{2g(H' - H)} = (m + m')\sqrt{2g(H'' - H)},$$

or

$$m\sqrt{H' - H} = (m + m')\sqrt{H'' - H}.$$

From the data obtained, compute each member of this equation, and compare.

The height of the two cylinders is best obtained by measuring the height of $A$ and of $B$ and averaging the two.

## EXPERIMENT 25

### TO DETERMINE THE ACCELERATION OF GRAVITY BY THE SIMPLE PENDULUM USING THE METHOD OF COINCIDENCE FOR DETERMINING THE TIME OF VIBRATION

**Apparatus.**—The pendulum consists of a metal sphere suspended by means of a fine wire attached to a crossbar forming a knife edge, and is shorter than a simple seconds pendulum. A point beneath the sphere just touches a small mercury surface when the pendulum is in the position of rest. The circuit of which the pendulum is a part contains a battery and a telephone receiver. At the end of every second the circuit is closed for an instant by means of a relay actuated by the laboratory clock and if the pendulum touches the mercury at this instant there will be a click in the receiver.

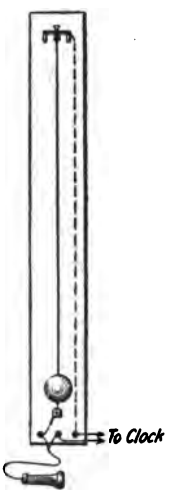


FIG. 23.

**Method and Manipulation.**—Determine the distance $l$ from the supporting knife edge to the center of the sphere. To do this measure length of the suspension from the

knife edge to the ball with a steel tape and the vertical diameter of the ball with a vernier caliper.

Cause the pendulum to swing through a small amplitude, by gently rocking the knife edge by hand and begin counting the swings. Record on the data sheet the counts corresponding to every click. Continue counting to five hundred or to any other number of the same order of magnitude. Repeat several times. Since the mercury surface has an appreciable width, the receiver will respond several times for each unison with the clock. The average of a set may be taken as the time of exact unison.

**Calculations.**—Arrange in a series the numbers representing the total number of swings (or half-vibrations) at each exact unison, as determined by averaging or interpolation as explained above. The pendulum observed is shorter than the standard pendulum and the latter requires exactly one second for each swing. Therefore the number of swings of the observed pendulum between any two points of exact unison, or $N$, is greater than the number of seconds between the same two points, or $T$. If the number of intervals of silence between the same two points is $p$, $T = N - p$, and since the time of one swing of the observed pendulum, or $t$, is given by the equation $t = T/N$, we have finally

$$t = 1 - p/N.$$

Note in the last expression for $t$ that $N$ is the only quantity determined by experiment that need be considered in deciding the number of significant figures in $t$, since $p$ must be an integer and cannot be in error by an entire unit. The smaller the fraction $p/N$, that is the more nearly the observed pendulum is the same length as the standard pendulum, the more accurately can $t$ be determined by this method.

Use several pairs of unisons from each series, choosing those that are separated by the same number of intervals of silence, so that all the values of $t$ to be averaged will be of the same degree of accuracy. Determine the acceleration due to gravity by computing $g$ from the formula

$$g = \frac{\pi^2 l}{t^2}.$$

Note that $t$ is half the period of vibration.

## EXPERIMENT 26

### ACCELERATION OF GRAVITY BY THE PHYSICAL PENDULUM METHOD

**Apparatus.**—A long rectangular bar, that has near one end an axis perpendicular to its plane. The bar is suspended in such a manner that the ends of the axis of suspension rest equally on the supporting surface at each side.

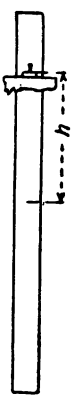


FIG. 24.

**Method and Manipulation.**—Obtain the time of vibration to an accuracy of 0.1 per cent.

Measure the length and width of the bar, also the distance from the upper end to the under side of the axis.

If $t$ = the time of vibration,
$I$ = the moment of inertia of the pendulum about the axis of suspension,
$h$ = the distance from the axis to the center of gravity of the pendulum,
$M$ = the mass of the bar,

then
$$t = 2\pi\frac{\sqrt{I}}{\sqrt{Mgh}}$$

and
$$g = \frac{4\pi^2 I}{Mht^2}.$$

Compute $I$ from the dimensions of the bar, and obtain $g$ by substituting the obtained data in the equation.

# FLUIDS

## EXPERIMENT 27

### SPECIFIC GRAVITY OF SOLIDS

**Apparatus.**—Specific gravity bottle; chemical balance; distilled water; the solid whose specific gravity is to be determined.

**Method.**—Obtain the mass $m$ of the substance. Use the greatest amount of the solid that can be placed in the bottle without interfering with its stopper. Fill the bottle with distilled water to a fixed point in the capillary tube. Wipe dry and obtain its mass $M$. Place the substance in the bottle and have the water extend to the same point in the capillary tube and obtain the mass $M'$. Air bubbles adhering to the solid must be removed.

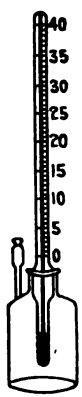


FIG. 25.

The specific gravity

$$s = \frac{m}{m + M - M'}.$$

**Precautions.**—The tube leading from the bottle must be closed to prevent evaporation. As the volume of the bottle and the density of the water change with temperature, care must be exercised to have the temperature of the water the same at each filling.

## EXPERIMENT 28

### SPECIFIC GRAVITY OF SOLIDS BY IMMERSION

**Apparatus.**—An analytical balance; distilled water; the solid whose specific gravity is to be determined;

a glass beaker and a stand for same which may be placed so that the beaker is over the left-hand scale pan; a fine wire for suspending the solid.

**Method and Manipulation.**—Determine the mass $M$ of the solid whose specific gravity is desired. Suspend the solid from the hook above the scale pan by means of the fine wire and determine the mass $M_1$ of the wire and solid in air. Place the beaker containing the distilled water on the stand and adjust so that the solid is immersed. Great care must be taken to remove any air bubbles adhering to the solid. Determine the apparent mass $M_2$ of the wire and solid. The specific gravity is

$$S = \frac{M}{M_1 - M_2}.$$

For more accurate determination the temperature of the water must be taken into consideration.

## EXPERIMENT 29

### TO DETERMINE THE SPECIFIC GRAVITY OF A LIQUID BY MEANS OF A MOHR'S BALANCE

**Apparatus.**—A Mohr's balance consists of a beam supported on steel knife edges. One half of the beam is graduated to tenths. Attached to the other half is a counter weight so that the beam will balance in air when a sinker $S$ is suspended at the end of the tenth division of the graduated arm. If the balance is imperfect it should be adjusted by the addition of a small weight (piece of paper) to the proper side before any readings are attempted. A rider is also provided which, when attached to the beam at the end of the tenth division, will cause the beam to balance when the sinker is immersed in water at 15° C. The volume of the sinker is five cubic centimeters, so the mass of the largest rider

is five grams. By Archimedes' principle, this rider is then equal in mass to the mass of the water displaced. Four other riders are also provided which have masses equal to 1.0, 0.1, 0.01, 0.001 of the mass of the rider mentioned. If the temperature corrections are not to be made the 0.001 rider will not be supplied. The sinker is provided with a thermometer so that the temperature at which it is used may be determined.

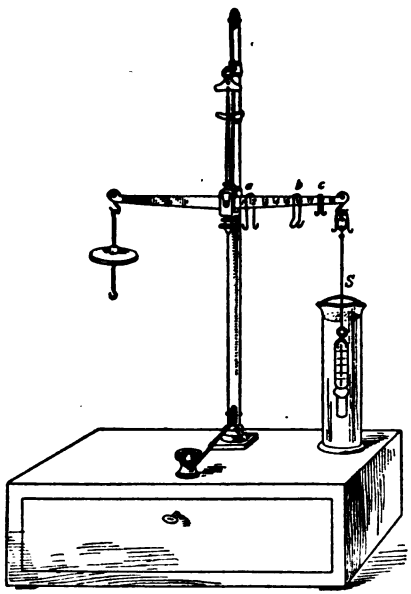



FIG. 26.

**Method and Manipulation.**—Suspend the sinker in the liquid the specific gravity of which is desired, and add riders until a balance is obtained. Then, if the riders are distributed as shown in Fig. 26, where *a* is the unit rider, *b* the 0.1 and *c* the 0.001, the specific gravity is 1.2608 if the temperature of the liquid is 15°C. If, however,

the temperature of the liquid is $t°$, and the reading of the balance is $w_t$, the density $\delta_t$ of the liquid is

$$\delta_t = \frac{\delta_{15} w_t}{w_{15}} \left(\frac{1 + \alpha 15}{1 + \alpha t°}\right),$$

and the specific gravity is

$$S = \frac{\delta_{15} w_t}{\delta w_{15}} \left(\frac{1 + \alpha 15}{1 + \alpha t°}\right),$$

where $\alpha$ = the coefficient of expansion of the sinker,
$\delta$ = the density of water at 4°C.,
$\delta_{15}$ = the density of water at 15° C.,
$w_{15}$ = the reading of the balance when sinker is immersed in water at 15° ($w_{15} = 1$ by construction of balance).

Since $\delta$ and $\delta_{15}$ are nearly equal, the above equation reduces to

$$S = w_t \left(\frac{1 + \alpha 15}{1 + \alpha t°}\right),$$

and since $\alpha$ is small we have approximately

$$\delta_t = S = w_t.$$

Use this equation for the approximate density required in Exp. 30.

The distribution of the riders on the graduated arm that causes the beam to balance when the sinker is immersed gives therefore the approximate value of the specific gravity of the liquid at any temperature.

Determine the approximate and the exact specific gravity of the provided solution.

## EXPERIMENT 30

### TO CALIBRATE AN HYDROMETER SCALE OF EQUAL DIVISIONS

**Apparatus.**—An hydrometer is an instrument for determining directly the density or specific gravity of a

liquid. It consists of a glass tube which is weighted down with mercury or lead at the lower end and which has a thinner portion at the other that is graduated in equal divisions or in unequal divisions that give directly the density or specific gravity.

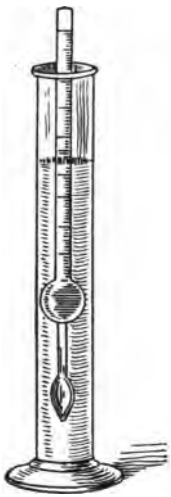


FIG. 27.

**Method and Manipulation.**—Prepare two solutions of such densities as will give a reading near each end of the scale when the hydrometer is immersed in them. Let $r_1$ and $r_2$ be the two readings obtained on the scale of equal divisions. Determine by means of a Mohr's balance the density $\delta_1$ and $\delta_2$ of the two solutions. Determine also the mass $m$ of the hydrometer to 0.1 gm. Then, since by Archimedes' principle $\frac{m}{\delta_1}$ and $\frac{m}{\delta_2}$ are the volumes of the liquid displaced in each case, the volume of the stem between $r_1$ and $r_2$ is $\frac{m}{\delta_1} - \frac{m}{\delta_2}$ and the volume per division is, if $\delta_1 < \delta_2$,

$$k = \frac{m}{r_2 - r_1}\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right).$$

Compute the value of $k$ in cubic centimeters.

Solving the above equation for $\delta_2$, and dropping the subscript in the case of $\delta_2$ and $r_2$, we obtain an equation which may be used in computing the density corresponding to any point $r$ on the scale, namely

$$\delta = \frac{m\delta_1}{m - k\delta_1(r - r_1)}.$$

Compute $\delta$ carefully for three or four values of $r$ so chosen that, taken together with the two points determined by the experimental values, the whole range of

the hydrometer will be covered and plot these values as ordinates and the scale divisions as abscissæ.

If a Baumé hydrometer is used compare the values thus obtained with the attached direct reading scale.

**Caution.**—Wash and dry the glass jar used with the Mohr's balance and the hydrometer itself after use with each solution, to avoid dilution or mixture of the standard solutions.

## EXPERIMENT 31

### SURFACE TENSION

If $h$ is the height to which a liquid rises in a capillary tube, $T$ the surface tension in dynes per centimeter length of film, and $r$ the radius of the tube, then

$$T = \frac{rh\rho g}{2 \cos \alpha}.$$

For water, since the density is (approximately) one gram per cubic centimeter, and the angle of contact on a clean glass surface is zero,

$$T = \frac{rhg}{2}.$$

**Apparatus.**—A glass tube; distilled water; a mirrored scale; a microscope with stage micrometer; cleaning solutions.

**Manipulation.**—After thoroughly cleaning the glass tube as directed below, heat it in a Bunsen flame until it becomes very pliable, then draw it into a capillary tube of about 0.5 mm. diameter. In order to make the cross-section of the capillary portion circular the tube must be heated evenly on all sides. Hold it horizontally with both hands, thumbs uppermost, and rotate it between each thumb and forefinger while heating. With-

draw it from the flame just before drawing out, and maintain some tension on the tube until the ends have cooled so much that there will be no bending at the thicker parts when the tension is relaxed. Break the tube so as to obtain a portion about 15 centimeters long and clamp it vertically so that it extends into a vessel filled to the brim with the liquid. Obtain the height $h$ by taking the reading $a$ on the scale for the surface of the liquid in the tube and likewise the reading $b$ for the surface of the liquid in the vessel.

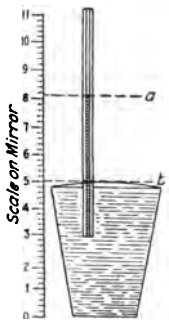


FIG. 28.

The diameter $2r$ of the capillary tube is measured by means of the stage micrometer. Cut the tube at the point of the water meniscus. To cut the tube at a desired point mark it *lightly* with a file and pull apart. Attach the tube to the stage by placing the piece to be measured in a slip of cork, with the cut end uppermost and focus the microscope so that the end is seen clearly. When the cross-hair is tangent to the bore, read the micrometer, then move the stage so as to make the cross-hair tangent to the other side, and again read. The microscopes supplied are so constructed that the microscope moves and the stage is stationary. Avoid back-lash, due to looseness of the micrometer screw, by always moving the microscope in the same direction up to the desired position. Measure two diameters at right angles to each other. Use the average of the two. From height $h$ and the diameter $2r$ compute the surface tension $T$.

**Precautions.**—The tubes will be found standing in a solution of chromic acid. Rinse with distilled water, dip in dilute nitric acid, rinse again with distilled water,

and dry by shaking. The tube need not be completely dry when heated for drawing, provided it be kept horizontal until cold, so that drops of water will not run down upon the hot glass, causing it to crack.

An alternative method is to use capillary tubes about 15 cm. long obtained from commercial stock of glass tubing. These are cleaned as directed above under "precautions." The diameter is measured by focusing microscope on end of tube.

# HEAT

## EXPERIMENT 32

### ATMOSPHERIC PRESSURE

The pressure of the atmosphere is measured by the weight of the column of pure mercury at 0° C. that it can support.

If $H_0$ is the height of the mercury column the pressure in dynes per square centimeter is

$$H_0 dg$$

where $d$ is in the density of mercury (13.596).

If the temperature is $t$, and if the reading $H$ for the height of the column is obtained by means of a scale that is correct at the temperature $t'$ the true height

$$H_t = [1 + \beta(t - t')]H,$$

where $\beta$ is the coefficient of expansion of the scale.

If the height of the mercury column is $H_t$ when its temperature is $t$, its height at zero degrees

$$H_0 = \frac{H_t}{1 + \alpha t},$$

where $\alpha$ is the coefficient of expansion of mercury.

The height of the column including both corrections is

$$H_0 = \frac{1 + \beta(t - t')}{1 + \alpha t} H.$$

If the scale is correct at 62° F. and $H$ and $t$ are obtained in English and Fahrenheit units, the equation becomes

$$H_0 = \frac{[1 + \beta(t - 62)]}{1 + \alpha(t - 32)} H,$$

where $\alpha = .0001001$ and $\beta = .00001043$ both per degree Fahrenheit.

The tension of the mercury vapor in the vacuum above the mercury causes a small depression, the correction for which may be obtained from tables. See Table 23.

The downward pull due to the meniscus of the mercury also causes a depression. This is reduced by using a tube of large diameter (25 mm.). The correction may be obtained from tables arranged for that purpose. Frequently the scale is shifted to offset this error.



FIG. 29.

**Apparatus.**—Fortin's barometer. The construction of this barometer is as indicated in Fig. 29. The scale attached to the metal casing is adjusted so as to measure from the end of the ivory point projecting down from the roof of the cistern. The mercury surface in the cistern can be raised or lowered by means of the screw underneath. The scale is provided with a vernier, and on the side opposite and connected to it is a slide whose lower edge is on a level with that of the vernier. By means of the head $c$ the vernier and slide may be moved up or down. A thermometer is attached to the metal casing of the barometer.

**Manipulation.**—Record the temperature, and adjust the surface of the mercury in the cistern so that it just touches the ivory point. Then adjust the venier so that the top of the meniscus is in line with the lower edges of

the venier and slide. Record the reading for the height, and correct it for expansion of scale and mercury by means of the equation given above.

Obtain correction from the table arranged for the barometer, and compare with corrected height. See Table 22 of Appendix.

## EXPERIMENT 33

### CALIBRATION OF A THERMOMETER

*The errors that this calibration is designed to correct are those due to the inaccuracies of the zero and boiling points on the thermometer scale and the irregularity of the bore of the tube. On account of these errors the thermometer scale does not accurately represent temperatures.*

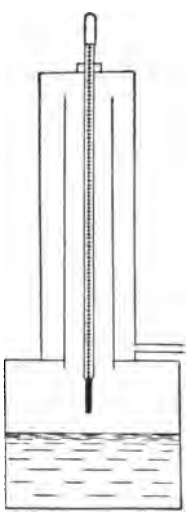


FIG. 30.

**Apparatus.**—Thermometer; barometer; funnel of finely divided ice; a boiler for holding the thermometer in steam.

**Manipulation.**—Immerse the thermometer up to the zero mark in melting ice. Set a small piece of mirror against the thermometer tube and view the image of the mercury thread in it, in order to avoid parallax.

After the mercury becomes stationary, note the reading $z$ on the thermometer scale, avoiding parallax by use of a mirror. Then place the thermometer in the boiler as shown in Fig. 30, so that the bulb is not less than two centimeters above the water surface. Allow the water to boil, and when the mercury surface becomes stationary, note the reading $a$ on the scale. Determine the barometric

height, and from it determine the temperature $A°$ at which water boils. The boiling temperature of water at 760 mm. (29.921 in.) pressure is 100° C. It decreases 1° C. for about 26.7 mm. (1.05 in.) decrease in pressure.

$$\therefore A° = 100° - \frac{760 - H}{26.7}.$$

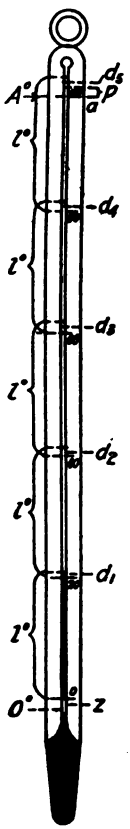



FIG. 31.

Separate a thread of mercury that is about 20 degree divisions long. This is best done by holding the thermometer in a horizontal position and giving it a light and quick tap at the upper end. If the thread that separates is not of the proper length join it to the rest and note the position on the scale where they come together. This, owing to a small air-bubble adhering to the glass, is in general the place where the mercury separates when the thermometer is again tapped even though mercury has been forced by the point. Then warm or cool the bulb until the desired length is above the joining point and tap. If unsuccessful, repeat the process. Having separated a thread of the desired length, place one end even with the zero on the scale, and read the fraction $d_1$ that the other end is above or below the 20 mark. Then place the lower end at 20 and read the difference $d_2$ at 40. Continue thus until the differences $d_3$ at 60, $d_4$ at 80, and $d_5$ at 100, have been obtained. Repeat by placing the upper end even with 100 and reading the difference at 80, etc.

Consider the quantities as indicated in Fig. 31.

Let $$100 - a = p.$$

Let $l^\circ$ = the number of degrees the temperature of the mercury must be raised to increase its volume an amount equal to the volume of the calibrating thread.

The markings on the thermometer tube are sufficiently accurate so that the difference between the length of one division on the scale and the actual length of expansion for one degree is so small that it may be neglected. Several divisions, however, do not represent the same number of degrees, for the differences accumulating make an appreciable error. Let the values obtained for the small distances $z$, $p$, and $d_1$. . . $d_5$ represent degrees. Then the number of degrees necessary to expand the volume of the mercury by the volume of the tube from 0° to $a$ is

$$z + l^\circ - d_1 + l^\circ - d_2 + l^\circ - d_3 + l^\circ - d_4 + l^\circ - d_5 - p$$

or

$$5l^\circ + z - d_1 - d_2 - d_3 - d_4 - d_5 - p.$$

But, experimentally, it is known that $A^\circ$ expands the mercury from 0° point to $a$.

$$\therefore A^\circ = 5l^\circ + z - d_1 - d_2 - d_3 - d_4 - d_5 - p,$$

and

$$l^\circ = \frac{A^\circ - z + d_1 + d_2 + d_3 + d_4 + d_5 + p}{5}.$$

As $A^\circ$, $z$, $\Sigma d$, and $p$ are known, $l^\circ$ becomes known.

If $z$ was above 0 on the scale, and if any value for $d$ represents a fraction below 20, 40, 60, 80 or 100, then their signs in the equation for $l$ and in the following equations must be changed.

The temperature for the division on the scale corresponding to the subscript of $t$ is as follows:

$$\begin{aligned}
t_0 &= +z^\circ, \\
t_{20} &= z + l^\circ - d_1, \\
t_{40} &= z + 2l^\circ - d_1 - d_2, \\
t_{60} &= z + 3l^\circ - d_1 - d_2 - d_3, \\
t_{80} &= z + 4l^\circ - d_1 - d_2 - d_3 - d_4,
\end{aligned}$$

$$t_{100} = z + 5l^\circ - d_1 - d_2 - d_3 - d_4 - d_5,$$
$$t_{100} = A^\circ + p \text{ (check)}.$$
$$t_1 = z + \frac{t_{20} - z}{20} = z + k,$$
$$t_2 = z + 2k,$$
$$t_3 = z + 3k,$$
$$t_{20} = z + 20k,$$
$$t_{21} = t_{20} + \frac{t_{40} - t_{20}}{20} = t_{20} + k',$$
$$t_{22} = t_{20} + 2k'.$$
etc.

Record the number of the thermometer used.

Compute the correction to be applied to the scale reading at 0, 20, 40, 60, 80, and 100, and plot a correction curve with the corrections as ordinates and scale readings as abscissæ. The smoothest curve that can be drawn so as to pass close to all the plotted points is the correction curve required. The thermometer calibrated is to be used in the heat experiments that follow.

## EXPOSED STEM CORRECTION

If it is desired to correct the temperature reading for the fact that a large part of the thermometer stem is at a lower temperature than the bulb, an auxiliary thermometer must be supported beside the first so that its bulb is at the same level as the center of the exposed portion of the mercury thread in the first. If

$T$ = the reading of the thermometer,
$Y$ = the point on its scale to which it is immersed,
$t$ = the reading on the auxiliary thermometer,
$Y$ = the coefficient of volume expansion of mercury (0.0001813 per °C.),
$a$ = the coefficient of linear expansion of glass (0.0000085 per °C.),
$T_c$ = the corrected temperature desired,

then $T_c = T + (T - Y)(T - t)(Y - 2a)$.

## EXPERIMENT 34.

### COEFFICIENT OF LINEAR EXPANSION OF A SOLID

**Apparatus.**—The solid is in the form of a tube about a meter long and about 0.8 cm. in diameter. This tube is placed inside another tube of about 5 cm. diameter which is packed with felt. Near each end of the small tube is a collar from which a bone rod projects at right angles and extends just through the side of the outer tube. At each end of the small tube is a trap containing a thermometer. A heavy iron block at each end supports the tube and a telescope. One of the telescopes

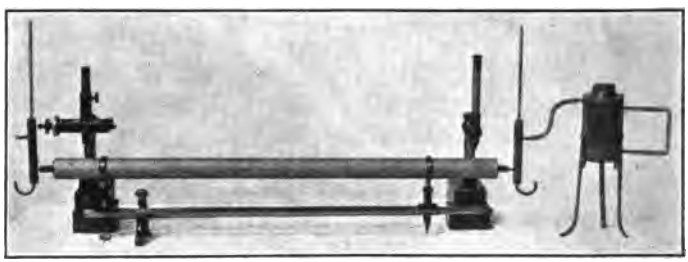


FIG. 32.

is provided with a stage micrometer. One of the supports for the tube carries a screw adjustment so that the tube may be moved in the direction of its length.

**Method and Manipulation.**—Pass a stream of ice water through the small tube. By means of a steel scale or tape measure the distance $L_1$ between the fine index lines on the bone projections. Focus the microscopes and adjust until the cross-hairs are over the index lines. Be careful to avoid back-lash on the part of the micrometer screw by approaching the index line in the direction in which the tube will expand. Obtain the micrometer reading $R_1$ and the readings $t_1$ and $t_2$ of the

thermometers. The thermometer readings $t_1$ and $t_2$ must be properly corrected. Their average will give the initial temperature $T_1$ of the tube. Pass steam through the small tube and when further expansion has ceased move the tubes by means of the screw support until the index line coincides with the cross-hair of the telescope which is not provided with the micrometer. Move the other telescope by turning the micrometer screw until the index line and the cross-hair coincide. Again obtain the micrometer reading $R_2$ and the temperatures $t_3$ and $t_4$ from which $T_2$ is obtained as above.

The coefficient of expansion is

$$a = \frac{R_2 - R_1}{L_1(T_2 - T_1)}$$

## EXPERIMENT 34*a*

### THE GAS LAW—BOYLE'S AND CHARLES' LAWS COMBINED

**Apparatus.**—The gas is enclosed in a glass tube, as shown at *A* in Fig. 32*a*. Inside this glass tube and projecting up through the mercury is a steel rod. This rod throughout its length has threads of one millimeter pitch, is finely pointed at its upper end, and has a micrometer head of one hundred divisions at the lower end. The mercury in contact with the gas in *A* communicates through a rubber tube with the mercury contained in a parallel glass tube *B*. The glass tube *A* is surrounded by a glass jacket filled with kerosene which may be heated by passing an alternating electric current through a coil of five turns of No. 24 IaIa wire wound on the stirrer *E*. The current may be obtained from the house circuit, using a bank of incandescent lamps *L* connected in series with the coil of the stirrer. As the lamps in the bank are in parallel, the current may be altered by con-

necting or disconnecting the various lamps. All metal parts coming into contact with mercury are of steel, and those that come in contact with the water in the jacket are of brass. All packings are of leather. The apparatus is supported by two parallel vertical rods 1.5 meters in height and having a tripod base.

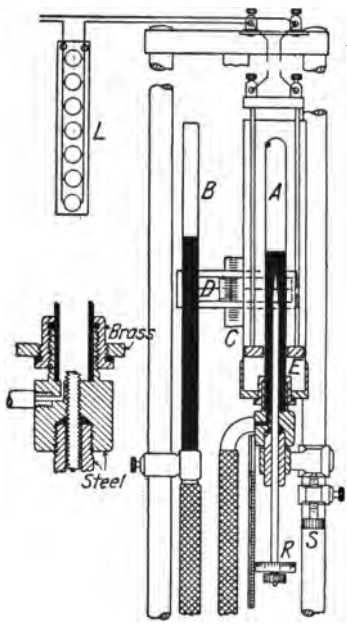


FIG. 32a.

READINGS.—The volume of the gas is obtained from the micrometer reading when the point of the rod is even with the mercury surface in $A$, an adjustment that can be made with great accuracy. The volume $V$ when the micrometer is at zero on the scale and the volume per unit length ($k$) are given. The pressure in centimeters of mercury height is the barometer reading plus the difference in heights of the mercury surface in $A$ and in $B$. This difference is obtained from the scale $C$. The slide $D$ carries a vernier and a mirror upon which is an index line which may be placed, free from parallax, even with the mercury surfaces or, in the case of the tube $A$, even with the lower end of the rod $R$. The temperature is obtained from a thermometer suspended in the liquid surrounding the tube $A$.

**Method and Manipulation.**—When Boyle's and Charles' laws are combined, the relation $\frac{PV}{T} = R$ (a

constant) is obtained, where $T$ is the absolute temperature, $P$ the pressure and $V$ the volume. When the gas in $A$ is at the room temperature determine its pressure, volume, and absolute temperature. Raise the temperature about five degrees by passing a current through the coil, being careful to adjust the current so that the temperature is constant while the readings are taken. In order to increase the range of pressure the tube $B$ may be raised arbitrarily. Determine again the pressure, volume, and absolute temperature of the gas. Continue thus until several sets have been obtained.

Compute the value of $R$ for each set of observations.

## EXPERIMENT 35

### THE RADIATING POWER OF DIFFERENT SURFACES

*The quantity to be determined is the number of calories lost per second, through radiation, by a unit area of a radiating surface when its temperature is one degree above the temperature of the surroundings.*

**Apparatus.**—The radiating body consists of a bright or black vessel $C$ filled with water. This vessel is suspended so as to be completely surrounded by a water jacket as shown in Fig. 33. The temperature of the radiating body and that of the surrounding surface are obtained by means of two thermometers placed as shown in figure.

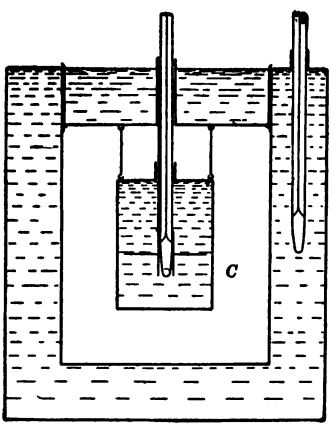


FIG. 33.

Thermometers reading 0.1° should be used.

**Method and Manipulation.**—Determine the mass and area of the vessel $C$ and fill it with water at about 60° C. Observe the temperatures of the water in the vessel $C$ and of the water in the surrounding jacket, at the end of every two minutes during a period of twenty minutes. In order to obtain a uniform temperature the water in $C$ and in the jacket must be stirred constantly. Always tap a thermometer that is indicating a decreasing temperature just before reading. Obtain the radiation curve by plotting the temperatures as ordinates and times as abscissæ. Likewise on the same sheet and with the same units obtain the curve showing the change in the temperature of the surrounding jacket with time. The scale of ordinates should be as large as the range of temperatures to be plotted in each case permits. Compute the coefficient of radiation at the instants 5, 10, 15 minutes in the following manner. Subtract the temperature $t'$ at 7.5 minutes from the temperature $t''$ at 2.5 minutes as obtained from the radiation curve. Using this value for $(t'' - t')$, compute the radiating power $k$ from the relation

$$k = \frac{(M + c + e)(t'' - t')}{TSt_a},$$

where $M$ = the mass of water in radiating vessel,
$c$ = the thermal capacity of the vessel,
$e$ = the thermal capacity of the thermometer, approximately equal to one calorie,
$t_a$ = the difference in temperature between the two cases at the instant selected for computing $k$,
$S$ = the area in sq. cm. of the radiating vessel,
$T$ = the time in seconds, i.e., 300.

Show the variation of these coefficients by plotting them as ordinates, using the corresponding values of $t_a$ as abscissæ.

Make the above determinations in the case of a bright and a black radiating vessel.

## EXPERIMENT 36

### THE ABSORBING POWER OF DIFFERENT SURFACES

*The quantity to be determined is the number of calories gained per second through radiation by a unit area of an absorbing surface when its temperature is one degree below the temperature of the surroundings.*

**Apparatus.**—Same as in Exp. 35.

**Manipulation.**—Fill the radiating vessel *C* with water at nearly 0° C., and proceed in the same manner as in Exp. 35.

## EXPERIMENT 37

### SPECIFIC HEAT

*The quantity to be determined is the number of calories necessary to raise the temperature of one gram of a given solid one degree.*

**Apparatus.**—The calorimeter *C* surrounded by a water jacket *B*; a steam jacket for heating the solid; two thermometers; a barometer; a watch.

**Method and Manipulation.**—Obtain the mass of the solid to be used, and place it in the inner chamber of the steam jacket. Pass steam through the outer chamber until the solid assumes the temperature of the steam. Obtain the mass of the calorimeter and fill it with water at room temperature to a point about two centimeters from the top. Obtain the mass of both water and calorimeter.

If there is to be a rise of more than five degrees have the temperature of the water correspondingly lower.

Stir the water constantly, and observe the temperature of the water in the calorimeter at the end of every minute for a period of about eight minutes. Immediately after the last reading, pass the solid quickly into the calorimeter. Stir, and record the temperature at the end of every thirty seconds until the temperature has reached a maximum, after that continue recording the

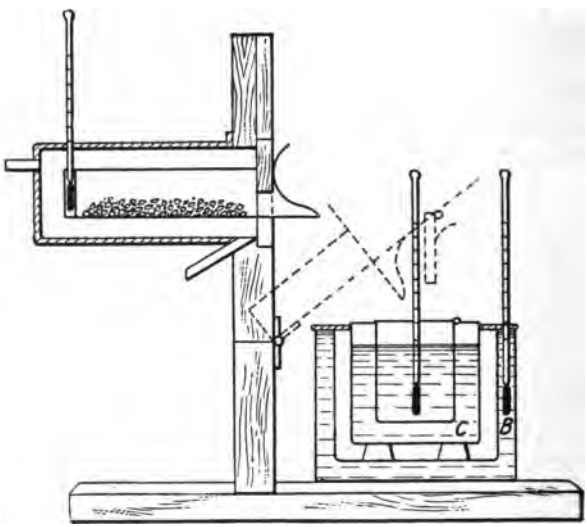



FIG. 34.

temperature at the end of every minute for an interval of about ten minutes. During the experiment the temperature of the water jacket must be recorded at short intervals. Record the temperature $t_s$ inside the steam jacket. This is best obtained by computation from the barometric height. (See Table 21.)

*Correction for heat loss:*

To obtain the corrected rise in the temperature of the water, plot the temperature readings on coördinate paper using times as abscissæ and temperatures as ordinates.

This curve has three regions, $AB$, $BC$, and $CD$. It is obvious that the temperatures represented by points on the portion $BC$ if above the temperature of the water jacket are low owing to heat losses by radiation, conduction, etc., and must be corrected in order to find the true rise in temperature of the water. These corrections may be obtained in the following manner:

From the portion $CD$ (see Fig. 34$a$) obtain the fall in temperature per second ($k$) by dividing the fall in tem-

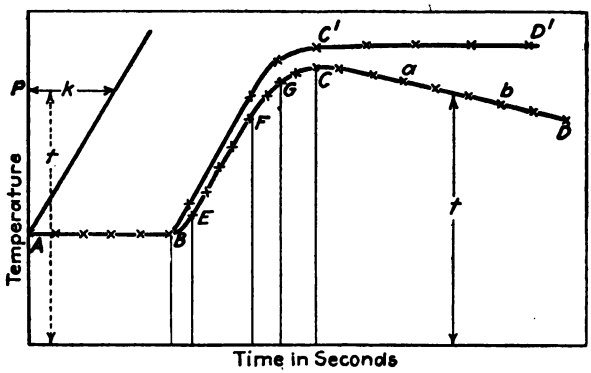



FIG. 34$a$.

perature in an interval ($ab$) of two or three minutes by the number of seconds in the interval. This may be taken as the rate of fall in temperature when the calorimeter is at the temperature ($t$) corresponding to a point at the middle of the interval used. From a point $P$ corresponding to $t$ on the axis of ordinates draw a line parallel to the axis of abscissas of a convenient length to represent $k$. From the point on the temperature axis representing the temperature of the water jacket, at the time of $t$, draw a straight line through the extremity of $k$. From this line the rates $k$ corresponding to any

temperature on $BC$ may be obtained, being simply by Newton's law of cooling the distance to the temperature axis. Divide $BC$ into segments over which the curve is straight or nearly so. These segments need not be of equal lengths. Obtain the rate $k_1$ for the temperature at the mid-point of the segment $BE$. Multiply $k_1$ by the time interval, $s_1$, of $BE$ in seconds. The amount $k_1 s_1$ added to the temperature of $E$ gives the corrected temperature. Proceed in the same way in connection with the segment $EF$. The correction to be added to the temperature of $F$ is the sum $k_1 s_1 + k_2 s_2$. Continue in this manner in connection with all the following segments. Add these corrections graphically to the corresponding temperatures and obtain the corrected temperature curve.

It is obvious that if the work has been correctly performed the upper part of the curve, namely $C'D'$, will be parallel to the axis of abscissas and represents the temperature, $t_m$, the water would have attained had there been no heat losses. The initial temperature $t_B$ may be taken as that of the point of intersection of $AB$ and $EF$ produced. The rise in the temperature of the water due to the heat obtained from the solid is therefore $t_m - t_B$.

It is to be noted that if the lower part of the curve is below the temperatures of the water jacket, the corrections are negative for the part of the curve which is below. It is desirable in this method to have the initial temperature of the water the same as the temperature of the water jacket provided, however, that the total rise does not exceed five degrees. If a greater rise than this is to be involved the initial temperature should be correspondingly lower. If the temperature of the water jacket is not constant care must be taken in finding $k$ to use the difference between the temperature

of the mid-point on segment and the temperature of the water jacket at the same instant.

If $M$ = the mass of the water,
$m$ = the mass of the solid,
$t_B$ = the initial temperature of the water,
$t_m$ = the final temperature of the water,
$t_s$ = the temperature of the steam or solid,
$c$ = the thermal capacity of the calorimeter,
$e$ = the thermal capacity of the thermometer,

then the specific heat

$$S = \frac{(M + c + e)(t_m - t_B)}{m(t_s - t_m)}.$$

From the values obtained for these quantities, compute the specific heat of the solid.

## EXPERIMENT 38

### LATENT HEAT OF FUSION

*The quantity to be determined is the number of calories necessary to change one gram of ice to water.*

**Apparatus.**—A calorimeter surrounded by water jacket; one thermometer; a watch.

**Method and Manipulation.**—Obtain the mass of the calorimeter and fill it with water at room temperature to a point about four centimeters from the top. Obtain the mass of the water and calorimeter, stir constantly, and record the temperature of the water at the end of every minute for a period of about five minutes. Then, quickly, after the last reading, drop a piece of dry ice of about 100 c.c. volume into the calorimeter so that it will be beneath the gauze stirrer. To insure the dryness of the ice keep it wrapped in cloth or absorbent paper until introducing it into the calorimeter. Record the tem-

perature at the end of every thirty seconds until the ice has melted, then continue taking the temperature at end of every minute for an interval of eight minutes. Likewise at the same time obtain the temperature of the water jacket. Continue stirring throughout the entire series of readings. Plot these readings and from the curve determine the fall in temperature of the water by the construction explained under Exp. 37. Weigh the calorimeter and water. The mass of the ice introduced is the difference between this and the mass previously obtained.

If $M$ = the mass of the water,
$m$ = the mass of the ice,
$t$ = the initial temperature of the water,
$t'$ = the final temperature of the water,
$e$ = the thermal capacity of the thermometer,
$c$ = the thermal capacity of the calorimeter,

then the latent heat of fusion

$$L = \frac{(M + c + e)(t - t') - mt'}{m}.$$

## EXPERIMENT 39

### LATENT HEAT OF VAPORIZATION

*The quantity to be determined is the number of calories necessary to vaporize one gram of water.*

**Apparatus.**—The apparatus is arranged as shown in Fig. 35. Steam is passed from the boiler into the trap and from there down into the immersed tank where it is condensed, and imparts its heat to the tank and surrounding water.

**Method and Manipulation.**—Obtain the mass of the calorimeter and of the coil. Fill the calorimeter with water sufficient to cover the coil, and add ice sufficient

to cool it 10 to 12° below the temperature of the water jacket. Obtain the mass of the water, and connect the coil and trap as shown in figure. Connection with the boiler is not made until the instant the steam is to be admitted. Stir the water, and record its temperature and the temperature of the room at the end of every minute during a period of about eight minutes. Immediately after the last reading allow the steam to enter. Stir, and record the temperature at the end of every thirty seconds, and when the temperature is as much above the room temperature as it was below at the beginning, disconnect the boiler. Continue recording the temperature at the end of every minute for a period of about eight minutes. The temperature of the water jacket must be recorded at frequent intervals. Wipe the coil dry, and weigh, and also obtain the temperature of the steam. (See Table 21.) Plot the temperature readings as ordinates and the corresponding times as abscissae. Obtain the rise in temperature by the method explained in Exp. 37.

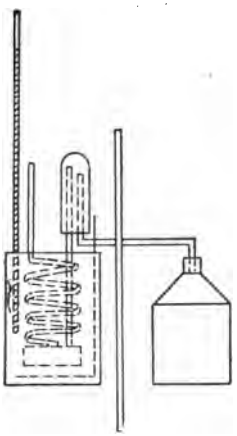


FIG. 35.

If $M$ = the mass of the water,
$M_1$ = the mass of the calorimeter,
$m$ = the mass of the coil and tank,
$m'$ = the mass of the steam condensed,
$t$ = the initial temperature of the water,
$t'$ = the final temperature of the water,
$t_s$ = the temperature of the steam,
$s$ = the specific heat of brass,
$e$ = the thermal capacity of the thermometer,

then the latent heat of vaporization

$$L = \frac{(M + M_1 s + ms + e)(t' - t) - m'(t_s - t')}{m'}.$$

**Precautions.**—The coil and tank must be dried before using by passing a current of warm air through them. Since the temperature of the water continues to rise after the steam has been cut off, disconnect the tube when the temperature is about one degree below the desired temperature.

## EXPERIMENT 40

### TO DETERMINE THE VAPOR PRESSURE OF WATER AT DIFFERENT TEMPERATURES

**Apparatus.**—An air tight boiler provided with a condenser. A tube leads from the condenser to a large vessel $V$ containing air, the purpose of which is to render the pressure steady. The vessel $V$ is connected to a manometer for determining the pressure. Pressures less than one atmosphere are obtained by exhausting the air from the boiler and vessel $V$ by means of a water pump connected to the tube $F$. The bulb of the thermometer is placed in oil contained in a closed tube extending into the boiler.

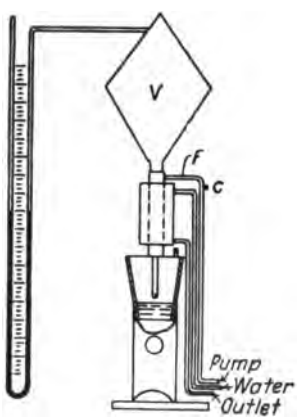



FIG. 36.

**Method and Manipulation.**—Obtain the temperature of boiling when the pressure is one atmosphere, then start the exhaust pump and reduce the pressure about 10

cm. Turn the stopcock $C$ so as to close the tube leading to the boiler. When the mercury in the thermometer becomes stationary, read the temperature and pressure. In case the apparatus is not quite air-tight the pressure will rise slowly after the pump is cut off. The minimum temperature should be taken and the manometer readings obtained at the same instant. The barometer must be read in order to compute the resultant pressure on the vapor in the boiler. This pressure should not be corrected for temperature unless the manometer difference is also corrected, and the latter is not measurable accurately enough to make this advisable. Then open $C$, and again reduce the pressure 10 cm. Close $C$, and when the thermometer becomes constant obtain the temperature and pressure. Continue thus until the pressure is as small as possible. Under the best conditions it is possible to boil the water in the apparatus supplied at a temperature less than 30°C.

Obtain the vapor pressure curve by plotting the pressures and the corresponding temperatures on coördinate paper, the temperatures being laid off along the axis of abscissas.

## EXPERIMENT 41

### TO DETERMINE THE RELATIVE HUMIDITY OF THE ATMOSPHERE

*The quantity to be determined is the ratio of the mass of the water vapor contained in a unit volume of air to the mass of water vapor it would contain if the air were saturated.*

#### 1. The dew-point method

**Apparatus.**—The dew point, or Daniell's hygrometer, consists of two glass bulbs joined by a glass tube as

shown in figure. The lower bulb has a bright metallic surface which enables the formation of dew to be more readily detected. This bulb also contains a thermometer. A thermometer for determining the room temperature is attached to the supporting stand. The instrument contains a quantity of ether sufficient nearly to fill one of the bulbs, no air being present.

**Method and Manipulation.**—In a vessel that is well surrounded by felt, place some water and finely divided ice, and in this immerse the upper bulb after the ether has been collected into the lower bulb. Record the air temperature $t_a$, and, when dew begins to form on the bright surface of the lower bulb, record the temperature $t_s$ of the metallic surface as indicated by the enclosed thermometer. Remove the ice and water, and note the temperature $t_s$ when the dew disappears. If the cooling is stopped as soon as the dew begins to form the two values of $t_s$ will not differ by more than a degree centigrade. Repeat several times, and obtain the average temperature $t_s$. This gives the temperature at which the amount of vapor present in the air is sufficient to saturate it.



FIG. 37.

Since the vapor present is open to the free air, its pressure does not appreciably change when its temperature is lowered. The pressure $p_a$ of the vapor present in the air is, therefore, the same as the vapor pressure of water at the dew point temperature and may be determined from the table of vapor pressures given in Table 24. The saturation pressure $p_s$ corresponding to the room temperature is obtained from the same table. From the gas law $pv = RmT$ we have the law that the

pressures are to each other as the masses. Therefore the relative humidity is

$$H = \frac{m}{m_s} = \frac{p_d}{p_s}.$$

From the values $p_d$ and $p_s$ calculate $H$.

### 2. The Wet and Dry Bulb or the Psychrometer Method

**Apparatus.**—Two thermometers are mounted side by side. The bulb of one is covered with a cloth which is kept saturated with water.

**Manipulation.**—Pass a current of air over the two bulbs of the thermometers when they are dry, by swinging the instrument about the handle in that direction

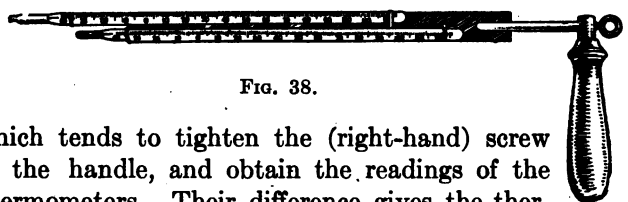


Fig. 38.

which tends to tighten the (right-hand) screw in the handle, and obtain the readings of the thermometers. Their difference gives the thermometer correction. Half of the correction is to be added to all readings taken with the thermometer reading the lower under these conditions and half to be subtracted from all readings taken with the other. Then saturate the cloth with water and again pass a current of air over the bulbs. Obtain the thermometer readings when they become steady and the barometer reading $B$. Correct the readings, and from the difference in the two thermometer readings, the reading $t_1$ of the wet-bulb thermometer, and the barometric pressure, obtain the vapor pressure $p_d$ from Table 25.

From Table 24 obtain the vapor pressure $p_s$ corre-

sponding to the reading of the dry-bulb thermometer. From the values $p_d$ and $p_s$ obtain the humidity $H$ as above.

A direct reading instrument on the psychrometer principle called a hygrodeik is also provided. Read the directions printed upon it, take readings on the two thermometers, and use the chart and swinging arm to find the relative humidity, the absolute humidity, and the dew-point. The instrument gives values that are too high unless placed in a strong current of air.

## EXPERIMENT 42

### THE MECHANICAL EQUIVALENT OF HEAT

*The quantity to be determined is the number of ergs equivalent to the energy of one calorie.*

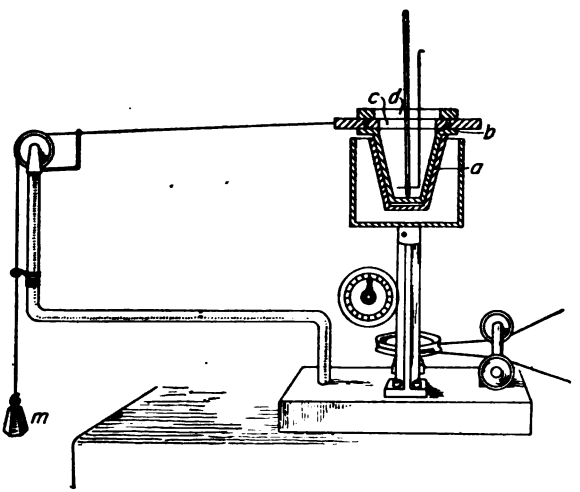



FIG. 39.

**Apparatus.**—A conical cup *a*, Fig. 39, is placed on ebonite supports inside a larger cup that is mounted

on a vertical axis. A friction cone $b$ fits the cup $a$ and supports a flanged wheel $c$, above which is a heavy metal ring $d$ that serves as a weight. A cord is wound about $c$, and, passing over a pulley, supports a weight $m$.

The friction cone $b$ is hollow and contains water, a thermometer, and a stirrer. A revolution counter is actuated by the vertical axis.

**Method and Manipulation.**—Fill the cup $b$ with a known mass of water, and let its temperature be about the temperature of the room. Do not fill the cup above the tapered portion. It is desirable to spend some time in so adjusting the friction that a fair-sized mass can be supported by it, since the best results are obtained when the rise in temperature is rapid.

This is secured by using the proper amount of oil between the cones. Record the reading of the revolution counter and obtain a series of readings at one minute intervals extending from several minutes before the rotation begins to several minutes after it stops. The water must be stirred constantly while the readings are being taken. Record again the reading of the revolution counter. Determine the corrected initial and final temperatures by the graphical method explained in connection with Exp. 37.

If $M$ = the mass of ($a$), ($b$), and stirrer,
$M_1$ = the mass of the water,
$m$ = the mass supported by the cord,
$s$ = the specific heat of the metal in $a$,$b$, and stirrer,
$n$ = the number of revolutions,
$t$ = the initial temperature of the water,
$t'$ = the final temperature of the water,
$e$ = the thermal capacity of the thermometer,
$r$ = the radius of the wheel $c$,

then the work done against friction is

$$2\pi rnmg \text{ ergs},$$

and the number of calories into which this work has been transformed is

$$(M_1 + Ms + e)(t' - t).$$

Therefore the mechanical equivalent

$$J = \frac{2\pi rnmg}{(M_1 + Ms + e)(t' - t)}$$

# ELECTRICITY AND MAGNETISM

## EXPERIMENT 43

### ELECTROSTATICS

**Apparatus.**—An electroscope; an ebonite rod; a glass rod in an ebonite handle; a large and a small piece of woolen cloth; a silk cloth; an insulated brass rod; a metal can; a proof plane; a piece of plate glass; a piece of sheet metal in a clamp.

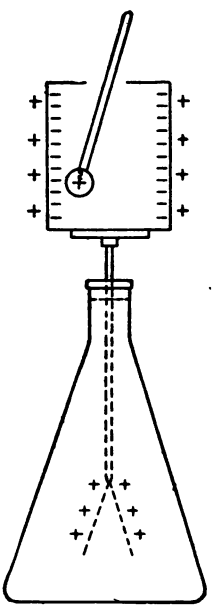


FIG. 40.

**Precaution.**—Special care must be exercised not to bring an excessive charge near the electroscope as it is liable to tear the leaves. Charge the rod at a distance, bring it slowly toward the electroscope, and stop when the leaves diverge the desired amount.

**Manipulation.**—*A.* 1. Charge the ebonite rod by rubbing it with a woolen cloth. Bring the rod near the uncharged electroscope, note effect and explain. Remove the rod, note effect and explain.

2. Repeat by using a glass rod rubbed with silk. Warm the rod to remove moisture, if it is necessary.

3. Rub the ebonite rod with wool and gather part of the charge on a proof plane. Bring the charged plane

into contact with the plate of the electroscope. Note effect and explain.

*B*. 1. Again charge the proof plane from the ebonite rod, and hold it near the plate of the charged electroscope. Explain.

2. Then rub a glass rod with silk, and hold it near the plate of the charged electroscope. Note effect and explain.

3. Discharge the electroscope. Rub the ebonite rod with wool and place it near the uncharged electroscope. Touch the plate of the electroscope with the hand. Remove the rod first. Explain.

4. Repeat and remove the hand first. Explain charging by induction.

5. Now bring the charged glass rod near the plate of the electroscope charged by operation *B*4. Explain.

6. Bring a charged ebonite rod near the plate. Explain.

7. Hold the hand or a sheet of metal near the plate of the charged electroscope. Explain.

8. Determine the kind of electrification on an insulated brass rod rubbed with wool. Explain why a divergence of the leaves rather than convergence is the test for electrification.

*C*. Place the metal can on the plate of the electroscope.

1. Tie the small piece of woolen cloth to one end of the glass rod. Hold the cloth-end of this rod and the ebonite rod in succession within the can. If either proves to have an electric charge, remove the charge by passing the rod rapidly through a Bunsen flame.

Hold the ends of the two rods within the can, and rub the cloth against the ebonite. Then remove the ebonite rod, and observe the leaves of the electroscope. Replace the ebonite rod, and withdraw the glass rod with the cloth. Observe the electroscope during the operation and explain.

2. Collect a negative charge on the proof plane and hold it inside the can without touching it, and move it about. Explain why the charge on the electroscope does not vary with the position of the proof plane as it does when the vessel is removed.

3. Again hold the charge on the inside of the can and touch the can with the hand. Explain. Remove the proof plane. Explain. Then touch the can with the charged proof plane. Does this prove that the induced and inducing charges are equal? What is Faraday's ice-pail experiment?

*D.* Remove the can, and charge the electroscope.

1. Test the piece of plate glass for electrification. If it is found to be charged, discharge it by passing it rapidly through a Bunsen flame.

2. Clamp a sheet of metal close to the plate of the electroscope and connect it to earth by holding the hand on the metal support or by means of a wire. Charge the electroscope and then note the effect of introducing the piece of glass between the sheet and the plate. Various insulating materials may be tested in the same manner as the glass.

Why must the metal sheet be connected to earth? What is an electric condenser? What is meant by specific inductive capacity?

3. Touch the plate of the electroscope with a proof plane. Observe the change on the separation of the leaves on removing the proof plane. Explain.

## EXPERIMENT 44

### TO PLOT A MAGNETIC FIELD

**Apparatus.**—Two magnets; a small magnetic needle mounted on a pivot; a large sheet of paper.

**Manipulation.**—Place the two magnets on the sheet

of paper 10 cm. apart, either parallel or perpendicular to each other, and so that a pole of one is equidistant from the two poles of the other. Mark dots on the paper about one centimeter apart around the edge of one of the magnets. Place the magnetic needle first at that end of the magnet from which the north-seeking or + pole is repelled having the center of the attracted or − end even with one of the dots, and mark with a dot the point on the paper that is even with the center of the repelled end. Then move the needle so that the center of the attracted end is even with this second dot, and continue as before until the second magnet, the edge of the paper, or the other end of the same magnet is reached. Draw a line through this succession of dots. This gives the direction in which the magnetic force moves the + pole of the needle and hence the direction of a magnetic line of force. Indicate this direction by means of an arrowhead. In a similar manner plot the magnetic lines of force from all the dots first placed around the magnet.

If any large portion of the second magnet is not reached by the lines of force coming from the first magnet, place dots a centimeter apart in this region, and plot as before.

Trace the outline of the magnets, and mark each pole with its proper sign.

Observe the direction of the magnetic lines of force in regions between poles that are attracting and that are repelling each other, also, the neutral point or points.

Observe also that "lines of force tend to become as short as possible and repel each other."

The direction of the lines is affected by the earth's magnetic field in regions where the field due to the magnets is weak.

## EXPERIMENT 45

### THE HORIOZNTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD—METHOD OF GAUSS I

**Apparatus.**—A magnetometer with agate cap needle; a bar magnet. A tangent galvanometer provided with a metric scale at right angles to the plane of the coil answers the purpose of a magnetometer. The fiber suspension of Exp. 46 is preferable wherever conditions make it possible to make accurately this magnetic measurement.

**Method.**—This absolute method for measuring the horizontal intensity of the earth's magnetic field consists of two independent determinations. The value of $\frac{M}{H}$ in terms of measurable quantities is obtained from the deflection produced on a short magnetic needle by a bar magnet, and the value of $MH$ from the time of vibration of this same bar magnet in the earth's magnetic field. Thus if $\frac{M}{H} = A$, and $MH = B$,

$$H = \sqrt{\frac{B}{A}}.$$

$M$ = magnetic moment of the bar magnet, and $H$ = the horizontal intensity of the earth's magnetic field.

#### 1. To determine $\frac{M}{H}$

**Method.**—The magnetometer is first adjusted so that the metric scale is at right angles to the direction

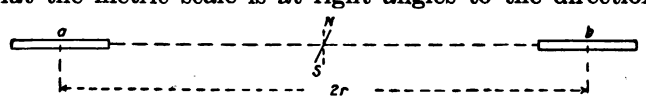



Fig. 41.

of the magnetic needle when the bar magnet is not acting. A deflection of the magnetometer needle is

then obtained when the bar magnet is in position $a$, Fig. 41. The magnet is then turned so as to reverse the position of the poles, and another deflection is taken. The bar magnet is then placed in position $b$, the same distance from the needle, and a similar set of observations taken. The angle of deflection is determined from the average of the scale deflections for the positions $a$ and $b$ of the magnet.

The distance $r$ is made large compared with the length of the needle. Thus in Fig. 42, $d \backsimeq r - l$. The force $f'$ with which the nearer pole of the magnet acts on a pole

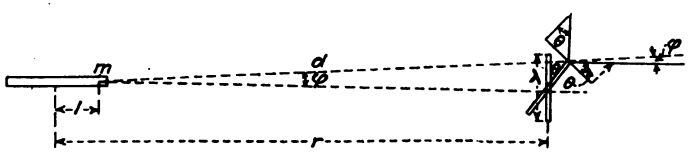



FIG. 42.

of the needle $= \dfrac{mm'}{(r-l)^2}$. The force $f''$ due to the other pole of the magnet $= \dfrac{mm'}{(r+l)^2}$ and acts in a direction practically opposite to that of $f'$.

$$\text{Thus } f = f' - f'' = \frac{mm'}{(r-l)^2} - \frac{mm'}{(r+l)^2} = \frac{4rlmm'}{(r^2-l^2)^2} = \frac{2Mm'r}{(r^2-l^2)^2}.$$

The component of this force tending to produce rotation $= \dfrac{2Mm'r}{(r^2-l^2)^2}\cos(\theta+\phi) \backsimeq \dfrac{2Mm'r}{(r^2-l^2)^2}\cos\theta$. As the other pole of the needle is acted on by an equal force, the moment due to this acting couple $\backsimeq \dfrac{2Mm'r\lambda}{(r^2-l^2)^2}\cos\theta$ where $\lambda$ is the distance between the two poles of the needle and $M = 2lm$. The horizontal component of

the earth's magnetic field acts on each pole of the needle with a force of $m'H$ dynes. The moment of this couple $= m'H\lambda \sin \theta$.

These two moments tend to turn the needle in opposite directions and since the deflected needle is at rest must be equal to each other. Then

$$\frac{2\,Mm'r\lambda}{(r^2 - l^2)^2} \cos \theta = m^1 H\lambda \sin \theta,$$

whence

$$\frac{M}{H} = \frac{(r^2 - l^2)^2}{2r} \tan \theta = A.$$

**Location.**—Both $\frac{M}{H}$ and $MH$ must be determined at the same location and where the tangent galvanometer of Exp. 55 is to be placed.

**Manipulation.**—It is best to use a magnet with a large magnetic moment and to take a large value for $r$, but in an elementary laboratory where several instruments are in use at the same time this condition must be modified. It is necessary, however, that $r$ be at least five times the length of the magnet and twenty times the length of the magnetic needle.

The distance $2l$ between the poles of the magnet is obtained approximately by means of a small compass or by assuming each pole to be one-twelfth the length of the bar from the end of the magnet. A considerable error in $l$ produces but a small error in the value of $\frac{M}{H}$. The distance $r$ is measured on the attached scale and the angle $\theta$ from the deflections.

### 2. To Determine $MH$

**Apparatus.**—The magnet used in the determination of $\frac{M}{H}$; a silk fiber attached to an aluminum saddle for

suspending the magnet; a box for excluding the air currents from the vibrating magnet; a circular scale; a watch.

**Method.**—The general equation for the time of vibration of any body vibrating about an axis with simple harmonic motion is $t = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\theta}{\alpha}}$ where $\theta$ is the angular displacement and $\alpha$ the angular acceleration for that displacement. In a magnetic pendulum suspended by a fiber with a negligibly small ratio of torsion the moment of the restoring couple

$$Fr = I_0\alpha = mH\lambda \sin\theta = MH \sin\theta.$$

From which

$$\frac{\sin\theta}{\alpha} = \frac{I_0}{MH}.$$

For small angles

$$\frac{\sin\theta}{\alpha} \fallingdotseq \frac{\theta}{\alpha}.$$

Then

$$t = 2\pi\sqrt{\frac{I_0}{MH}},$$

and

$$MH = \frac{4\pi^2 I_0}{t^2},$$

where $I_0$ is the moment of inertia of the magnet and $t$ the time of a double vibration.

**Manipulation.**—The magnet is suspended at its center of gravity in the location occupied by the needle of the magnetometer in the determination of $\frac{M}{H}$. The magnet is made to vibrate in an arc of about 30° and care is taken to have the magnet free from any pendular motion. The time of vibration is determined in the ordinary manner. When $H$ is to be determined to

within 0.1%, the time of the vibration should be determined to within 0.05%. The moment of inertia of the magnet can be determined experimentally or calculated. The magnet used is usually a cylinder, the moment of inertia of which, when rotating on an axis perpendicular to its center, is $M'\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$ where $L$ is the length of the cylinder, $R$ its radius, and $M'$ its mass.

Since the expression for the time of vibration is correct only for very small angles, the comparatively large angle used necessitates a small correction to the observed time. When $t$ is the observed time, and $\theta$ the average complete arc of vibration, the corrected time

$$t_0 = t\left(1-\tfrac{1}{4}\sin^2\frac{\theta}{4}\right)^1.$$

## EXPERIMENT 46

### THE HORIZONTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD—METHOD OF GAUSS II

**Apparatus.**—A magnetometer with fiber suspension; a reading telescope for mirror-scale method of measurement; a bar magnet.

**Method.**—The method is the same as that of Exp. 45 except that a correction for the torsion of the fiber is made. The restoring moment is now due both to the earth's magnetic field and to the torsion of the twisted fiber and equals

$$m'H\lambda\ (1+\Theta)\ \sin\ \theta$$

as is shown in Exp. 50.

Then

$$\frac{2\,Mm'r\lambda}{(r^2-l^2)^2}\cos\theta = m'H\lambda(1+\Theta)\sin\theta,$$

[1] Ziwet, Part 1., p. 121.

whence

$$\frac{M}{H} = \frac{(r^2 - l^2)^2}{2r}(1 + \Theta) \tan \theta.$$

**Adjustment.**—1. Place the magnetometer on the shelf in the exact designated position. The value of the horizontal component of the earth's magnetic field varies considerably in different parts of the room due to the presence of steel beams and iron radiators. It is for this reason that the magnetometer is placed at some definite point.

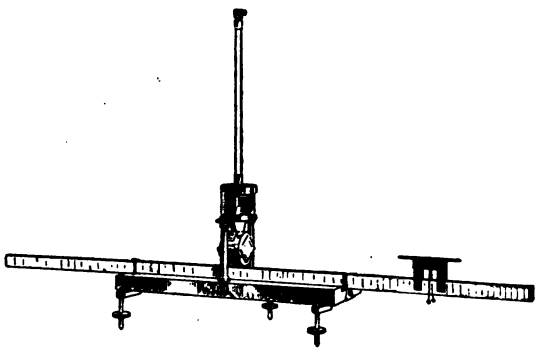


FIG. 43.

2. Raise the needle by turning the upper attachment of the suspension fiber a fraction of a turn while watching the needle, and adjust the level of the magnetometer until the needle is free.

3. Turn the magnetometer until the horizontal scale is in the magnetic east and west, i.e., perpendicular to the magnetic meridian. The plane surfaces of the cylindrical copper damping block in which the annular magnetic needle hangs are perpendicular to the scale. When the plane of the needle is parallel to a plane of this surface or to the plane separating the two halves of the copper block, the scale is perpendicular to the magnetic

meridian. In laboratories where there are no appreciable variations in the direction of the earth's field as modified by the magnetic materials of the building, the adjustment can be more accurately made by means of a compass attached to a brass bar.

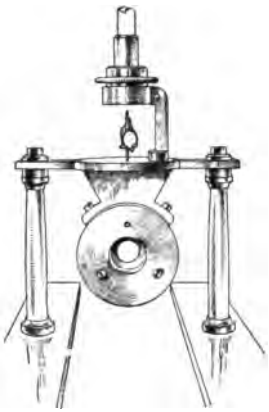


FIG. 44.

**Manipulation.**—Earth currents due to electric street cars, etc., cause the magnetic needle to vibrate. The position of the needle, in such a case, is determined by taking the mean of its extreme positions, the large accidental variations not being taken into consideration, or by using the average of many observations taken at regular intervals. This disturbance is usually not noticed when using the less sensitive instrument of Exp. 45.

For purposes of reference the various operations or measurements are described under the four succeeding experiments.

## EXPERIMENT 47

### ADJUSTMENT OF TELESCOPE FOR THE TELESCOPE-MIRROR-SCALE METHOD OF MEASUREMENT

**1. Position of Telescope.**—Place the telescope stand at a distance of 100 to 150 cm. in front of the magnetometer mirror, and raise the stand until a perpendicular to the mirror passes through a point half-way between the axis of the telescope and the reading scale. When the observer sees the image of his eye in the mirror, his eye is in the perpendicular to the mirror. This adjustment is often most readily made by holding the eye on the level of the telescope and raising or lowering the stand until the image of the scale is seen in the mirror.

**2. Focusing of Cross-wires.**—Adjust eye-piece of the telescope until the cross-hairs are seen most distinctly.

**3. Final Adjustment of Height of Telescope Stand.**—Focus the telescope upon the image of the scale in the mirror. If the image of the scale cannot be found, the telescope stand is usually too high, or too low, or the telescope is not pointed properly. Adjust the height of the stand until the scale divisions appear in the center of the field.

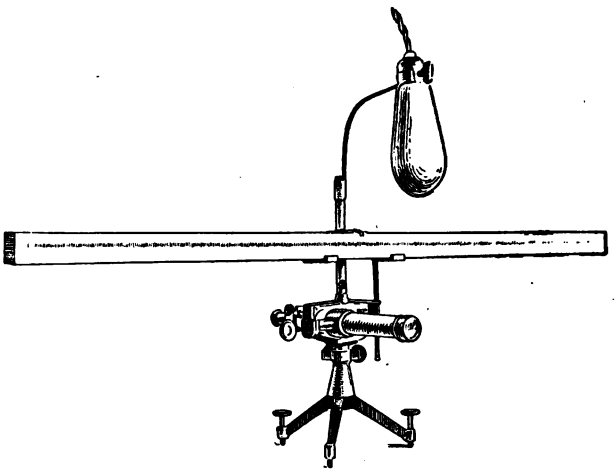


FIG. 45.

**4. Position of Reading Scale.**—The scale must be placed perpendicular to the telescope.

**5. Final Adjustment for Horizontal Position of Telescope.**—Move telescope stand to the right or to the left until the point on the scale directly above the axis of the telescope appears on the vertical cross-hair of the telescope. This point on the scale is called the *null point* or the *zero point.*

**6. Parallax.**—When the eye-piece is adjusted so that the cross-hairs are seen most distinctly, and the tele-

scope is adjusted until the image of the scale is seen most clearly, there should be no parallax, i.e., the image in the telescope and the cross-hairs should be in the same plane, so that when the eye is moved from side to side of the eye-piece, the vertical cross-hair does not change its position with respect to the scale. If the reading changes as much as 0.1 of a mm. with the motion of the eye, change adjustment of the telescope or of the eye-piece for the cross-hair, or of both, until there is no appreciable parallax.[1]

## EXPERIMENT 48

### MEASUREMENT OF ANGLES OF DEFLECTION BY THE TELESCOPE-MIRROR-SCALE METHOD

When the null point of the scale is seen on the cross-hair of the telescope, a ray of light coming from the null point of the scale, *O* in Fig. 46, is reflected from the mirror

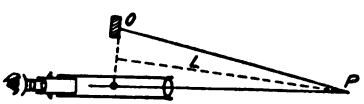



FIG. 46.

at the point *P*. The incident and reflected rays are in the same vertical plane perpendicular to the mirror as shown in Fig. 47. When the mirror is turned through the angle $\theta$, this vertical plane moves from *PO*, Fig. 47, to *Pd*, so that a ray of light coming from *S* in a vertical plane *SP*, will be reflected in the vertical plane *OP* making the angle $SPO = 2\theta$.

If *D* be the observed deflection, *OS*, and *L* be the per-

[1] The Telescope-Mirror-Scale Method, Adjustments and Tests, by S. W. Holman, 1898.

pendicular distance between the plane of the mirror and the plane of the scale, then

$$\tan 2\theta = \frac{D}{L}.$$

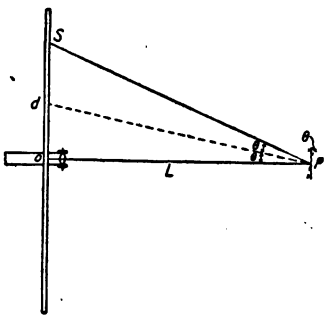



FIG. 47.

The value of $2\theta$ is obtained from a table of tangents, and $\theta$ from the determined value of $2\theta$.

When a circular scale is used,

$$2\theta = \frac{D}{L}.$$

## EXPERIMENT 49

### TO REMOVE TORSION FROM MAGNETOMETER FIBER

**Apparatus.**—Magnetometer of Exp. 46 and telescope of Exp. 48.

**Manipulation.**—By means of a pencil or a magnet carefully turn the magnetic needle 360° in one direction and after the needle comes to rest note the deflection. Turn the needle back to its original null position, take reading and then turn it 360° in the opposite direction, take deflection reading and again the null reading. If the two deflections are equal there is no torsion in the fiber when the needle is in its original position. If the two deflections are not equal, use that position in which it gave the greater deflection[1] as a *new original position*, adjust the telescope again to give the reading of the null point, and then find the two deflections as above. Continue in this manner until the two deflections are equal. If it is necessary to remove torsion of less than one turn,

[1] This position is used in case a cocoon fiber is employed. For a perfectly elastic fiber the position giving the smaller deflection would be taken.

the tube which holds the upper support of the fiber must be turned a fraction of a revolution at a time, and tests for torsion made until it is removed. All readings must be taken as soon as possible after the needle comes to rest, for the large twist given the fiber gradually causes it to obtain a set. To partly correct for the set in the fiber take a reading at the null position of the needle after each observation, and use this in computing that deflection.

## EXPERIMENT 50

### RATIO OF TORSION OF A MAGNETOMETER FIBER

**Apparatus.**—A magnetometer adjusted as in Exps. 46, 47, and 49.

**Method.**—The magnetometer needle has the direction of the magnetic meridian when its supporting fiber hangs without torsion. If it is deflected from this position, the fiber becomes twisted and exerts a force tending to bring the needle back into the meridian. Both the earth's field and the torsion of the fiber thus tend to move the needle into its original position. The moment of the magnetic force, $Hm\lambda \sin \theta$, varies as the sine of the angle of deflection, and the torsional moment, $T\theta$, as the angle itself. Since the angle of deflection used is small, its sine may be considered equal to the angle; thus as the angle increases the moment of the torsional couple increases at the same rate as that of the magnetic couple. The value of the former may than be expressed as a fraction of the latter. This fraction is called the *ratio of torsion* of the fiber and is represented by the letter $\Theta$.

The total moment of the forces tending to restore the needle into its original position $= Hm\lambda \sin \theta + T\theta = Hm\lambda \sin \theta \left(1 + \frac{T\theta}{Hm\lambda \sin \theta}\right) = Hm\lambda(1 + \Theta) \sin \theta,$

where $H$ = horizontal intensity of the earth's field,
$m$ = pole strength of the needle,
$\lambda$ = distance between the poles of the needle,
$\Theta$ = ratio of torsion,
$\theta$ = angle of deflection,
$T$ = moment of torsion at unit angle.

To measure $\Theta$ turn the needle through 360°. The fiber is then twisted and turns the needle back a fraction of a revolution until the moment of its torsional couple equals the moment of the magnetic couple. This fraction of a revolution is the angle $\phi$ of Fig. 48. The fiber is

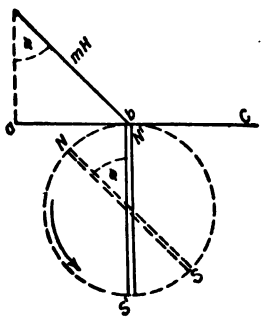



FIG. 48.

twisted through $(360° - \phi)$; hence the moment of the torsional couple $= (2\pi - \phi)T$. The moment of the magnetic couple at the same time $= Hm\lambda \sin \phi$. Since the needle is at rest and $\phi$ is small,

$$(2\pi - \phi)T = Hm\lambda \sin \phi \doteq Hm\lambda \phi.$$

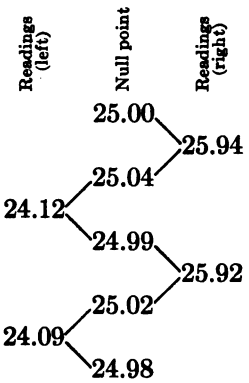
$$\therefore \frac{T}{Hm\lambda} \doteq \frac{\phi}{2\pi - \phi}.$$

But

$$\Theta = \frac{T\theta}{Hm\lambda \sin \theta} \doteq \frac{T}{Hm\lambda}.$$

$$\therefore \Theta = \frac{\phi}{2\pi - \phi} = \frac{\phi^\circ}{360^\circ - \phi^\circ}.$$

**Observations.**—After torsion is removed from the fiber, a set of scale readings is taken from which the ratio of torsion is to be computed. One method for recording the observations is here given as an illustration:




The perpendicular distance from the mirror to the plane of the scale is measured.

**Calculations.**—The null point taken after a deflection is the one employed in computing the magnitude of each deflection.

The average of the several approximately equal deflections is taken for computing the value of $\phi$ which is obtained in the manner explained in Exp. 48.

## EXPERIMENT 51

### THE MAGNETIC MOMENT AND THE POLE STRENGTH OF A MAGNET

**Method.**—From Exp. 45 or Exp. 46 the values of $\frac{M}{H}$ and $MH$ are known for the magnet employed.

$$\frac{M}{H} = A, \quad MH = B, \quad \text{then } M = \sqrt{AB}.$$

But $$M = 2lm.$$

$$\therefore\ m = \frac{\sqrt{AB}}{2l}.$$

Determine also the horizontal force with which each pole of the magnet was acted on in the location where $H$ was determined.

## EXPERIMENT 52

### TO DETERMINE H BY COMPARISON

**Method.**—The time of vibration of a magnet is found in a place where $H$ is known, and then where $H$ is to be determined.

$$t = 2\pi\sqrt{\frac{I}{MH}},\quad t' = 2\pi\sqrt{\frac{I}{MH'}}.$$

$$\therefore\ \frac{t}{t'} = \sqrt{\frac{H'}{H}},\ \text{and}\ H' = \frac{t^2}{t'^2}H.$$

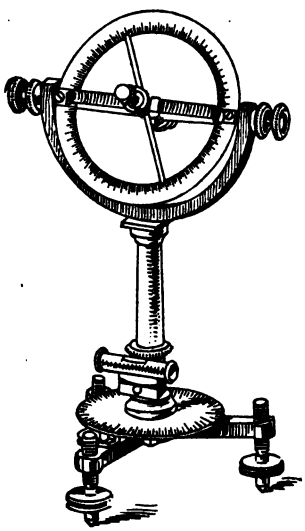


FIG. 49.

## EXPERIMENT 53

### THE INCLINATION AND THE INTENSITY OF THE EARTH'S MAGNETIC FIELD

**Method and Manipulation.**—The inclination compass, Fig. 49, is used to measure the inclination or angle of dip. It consists of a magnetic needle having a pivot passing through its center of gravity and attached at the center of a graduated circle. This circle can be rotated about a horizontal axis, and its support about a vertical axis.

The circle is placed in a horizontal plane so that the needle takes the direction of the magnetic meridian. The compass is then turned until the direction of the axis of rotation of the graduated circle coincides with that of the needle. The circle is now turned into a vertical position. Its plane then is in the magnetic meridian, and the magnetic needle takes the direction of the magnetic field. Readings are taken at both ends of the needle, from which the inclination is determined.

Knowing the horizontal intensity from Exp. 45 and the angle of inclination, the intensity of the earth's field can be calculated from $F = \frac{H}{\cos \theta}$.

**Sources of Error.**—1. The axis of suspension of the needle may not pass exactly through its center of gravity.

2. The geometrical and magnetic axes of the needle may not coincide.

## EXPERIMENT 54

### ADJUSTMENT OF A TANGENT GALVANOMETER

**Apparatus.**—Tangent galvanometer; battery of constant electromotive force; commutator; adjustable resistance.

The tangent galvanometer consists of a vertical coil of wire having a short magnetic needle at its center. If the length of the needle is less than one-twelfth the diameter of the coil that part of the field in which the needle swings may be considered uniform and of the same intensity as the field at the center of the coil. [1]

**Adjustment.**—1. After placing the galvanometer in the location where the horizontal component of the earth's magnetic field has been determined, free the needle in the same manner as in the case of the magnetometer.

[1] Nipher, pp. 162–166.

2. Turn the galvanometer until the plane of its coil is in the magnetic meridian as nearly as it is possible by direct observation of the direction of the free magnetic needle and the parts of the body of the galvanometer.

3. If the needle is suspended by a fiber remove the torsion and determine the ratio of torsion as in Exps. 49 and 50.

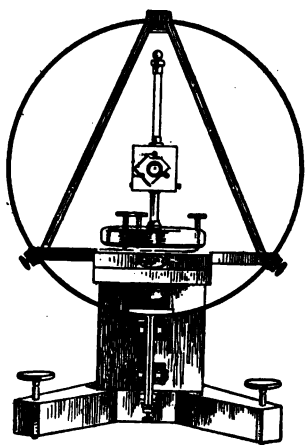


FIG. 50.

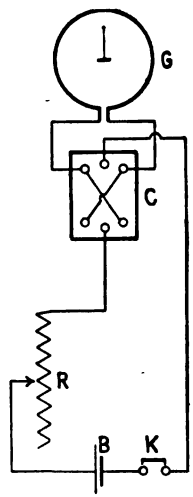



FIG. 51.

4. Repeat adjustment 2.

5. Where the galvanometer needle is not disturbed by earth currents it is possible to make a more accurate adjustment of the plane of the galvanometer coil than the approximate adjustment given above. The galvanometer is connected to the accessory apparatus as shown in Fig. 51.

The electric current is regulated by means of the adjustable resistance $R$ until it gives a large deflection on the scale. It is then reversed by means of the

commutator $C$ and the deflection on the other side of the null point is observed. If the two deflections are equal the coil is parallel to the needle, and is in the magnetic meridian. If the deflections are not equal, the galvanometer is turned so that the plane of its coil approaches the direction of the needle when in the position of its smaller deflection. Continue in this manner until the deflections are equal. If the current changes with time the two deflections must be taken in rapid succession.

## EXPERIMENT 55

### TO DETERMINE THE CONSTANT OF A TANGENT GALVANOMETER FROM THE DIMENSIONS OF ITS COIL

**Apparatus.**—The tangent galvanometer of Exp. 54.

**Magnetic Field About a Wire Carrying an Electric Current.**—The intensity of the magnetic field is proportional to the current, and, for each element of the current, varies inversely as the square of the distance from the wire.

**Electromagnetic Unit of Current.**—If a conductor is bent in a loop of one centimeter radius, the current through it has unit strength when for each centimeter of the conductor a force of one dyne is exerted on a unit magnetic pole at the center. Whence at the center of a circular loop consisting of $n$ turns of wire the intensity of the field $F = \frac{2\pi r n I'}{r^2} = \frac{2\pi n I'}{r}$, where $I'$ is the strength of the current in electromagnetic units.

The force acting on a pole of $m$ units at the center is

$$Fm = \frac{2\pi n m I'}{r}.$$

**Practical Unit of Current.**—The *ampere* is 0.1 of the electromagnetic unit of current.

**Method.**—Fig. 52 represents a horizontal section of the galvanometer. The earth's field acts on each pole of the needle with a force of $mH$ dynes. The field due to the current acts with a force of $\frac{2\pi nmI'}{r}$ dynes in a direction perpendicular to the former force.

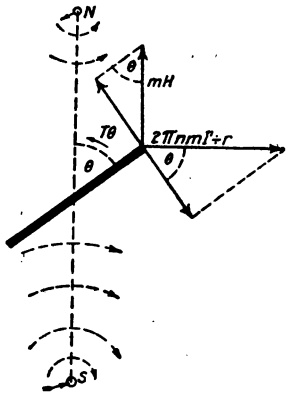


FIG. 52.

When the needle is not suspended by a fiber these are the only two forces acting that can produce rotation. Their components at right angles to the needle act in opposite directions and the magnitude of one increases while that of the other diminishes as the angle of deflection increases. The needle comes to rest when the angle of deflection is that at which these two components, or their moments, are equal to each other.

Then considering both poles

$$\frac{2\pi nm\lambda I'}{r}\cos\theta = mH\lambda\sin\theta,$$

$$I' = \frac{Hr}{2\pi n}\tan\theta,$$

$$I = 10I' = \frac{5Hr}{\pi n}\tan\theta = K\tan\theta,$$

where

$$K = \frac{5Hr}{\pi n}, \text{ a constant.}$$

If the needle is suspended by a fiber the tortional couple due to the twisting of the fiber tends to move the needle in the same direction as does the earth's field.

Then

$$\frac{2\pi nm\lambda I'}{r_1}\cos\theta = mH\lambda\sin\theta + T\theta = mH\lambda(1+\Theta)\sin\theta,$$

$$I' = \frac{Hr(1+\Theta)}{2\pi n}\tan\theta,$$

$$I = 10I' = \frac{5Hr(1+\Theta)}{\pi n}\tan\theta = K\tan\theta,$$

where

$$K = \frac{5Hr(1+\Theta)}{\pi n}, \text{ a constant.}$$

**Manipulation.**—If a pivot compass is used the friction of the agate bearing may influence the position of the needle. Tapping the compass lightly with a pencil diminishes the error due to this effect. The value of $H$ is determined by Exp. 45 or Exp. 46 and $r$ is measured. Calculate the constant for each of the coils on the galvanometer and for all possible combinations of them.

## EXPERIMENT 56

### TO CALIBRATE AN AMMETER BY MEANS OF A TANGENT GALVANOMETER

**Apparatus.**—An ammeter; a battery; a tangent galvanometer; a rheostat; a key, and a commutator.

The essential part of an ammeter of the Weston type consists of a coil of wire held by two spiral springs and pivoted on an axis in the magnetic field between the poles of a permanent magnet, as shown in Fig. 54. The deflection of this coil, which is indicated by an attached pointer on a graduated scale, as shown in Fig. 53, depends on the strength of the current flowing through it. The scale is originally graduated by passing known currents through the ammeter and marking the points to which the pointer is deflected. The strength of the magnet changes with time and usage and

thus, for accurate work, the ammeter must occasionally be calibrated.

*Great care must be exercised always to connect the + pole of the battery to the binding post similarly marked and*




FIG. 53.

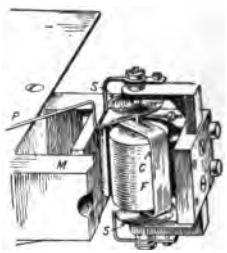



FIG. 54.

*never to send a current through the instrument that produces a deflection greater than the full reading of the scale.* Before connections are made to the instrument, put in the circuit all of the resistance on the rack provided and diminish this gradually until the desired current is obtained.

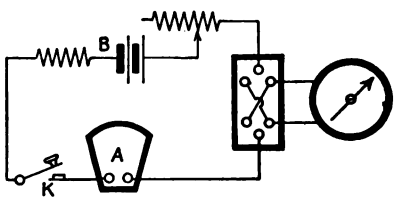



FIG. 55.

**Method and Manipulation.**—Pass a current through the ammeter, tangent galvanometer and rheostat in series as shown in Fig. 55. In connecting the apparatus, care must be taken to have the commutator reverse the current through the tangent galvanometer only, for the current through the ammeter must always be in the

same direction. Measure the current by means of the tangent galvanometer and record the corresponding reading of the ammeter. Tabulate the results for several readings which extend at approximately regular intervals over the full range of the scale.

In reading the ammeter, care must be taken to avoid parallax and to take the "zero" reading. The value of the current as read on the scale of the ammeter must be corrected by + or − the value of the zero reading. Always record the zero reading.

The deflections on the tanget galvanometer must be large so that the observational error in the determination of the value of the current is less than the observational error in the corresponding ammeter reading. If, as is nearly always the case, the percentage of error in the readings of the ammeter is practically the same for all the readings, this percentage with its proper sign, + or −, is called the *correction factor* of the ammeter. Determine this correction factor for the instrument calibrated. The correction factor is + if it is to be added to the ammeter reading.

In laboratories where stray earth currents affect the readings of the tangent galvanometer only a rough calibration can be made. Results, however, should be obtained having as high a degree of accuracy as is possible under the existing conditions. A more convenient method of calibration is described in Exp. 82.

## EXPERIMENT 57

### THE CORRECTION FACTOR OF AN AMMETER—SILVER VOLTAMETER METHOD

**Apparatus.**—Silver voltameter; silver nitrate solution; ammeter; key; resistance rack; battery.

**Method.**—Accurate determinations have been made of the mass of silver deposited by a known current from

a solution of silver nitrate.[1] It has been found that the deposit for any given time is proportional to the current and that 0.001118 gram is deposited by one ampere in a second. Hence, the average strength of any current is

$$I = \frac{m}{0.001118t},$$

where $m$ is the mass of silver deposited and $t$ the time.

**Manipulation.**—The voltameter $V$ and ammeter $A$ are connected to the battery as shown in Fig. 56. The cup of the voltameter is the cathode and is connected to the negative pole of the battery. The voltameter, before filling with $AgNO_3$, is short-circuited and the resistance is adjusted until a conveniently large deflection is obtained on the ammeter. The short-circuit is then removed and the circuit is opened. The anode is covered with filter paper or is placed within a porous cup to prevent particles dropping from it into the cathode cup, and is so adjusted as to be at the approximate center of the latter. The cups are then filled with the silver nitrate solution and the circuit is closed, the time of closing being accurately noted. The current is allowed to flow about an hour and is kept constant by occasional slight changes in the adjustable resistance. Finally, when the circuit is opened, the time is again noted.

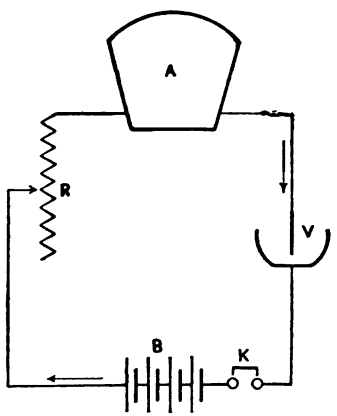


FIG. 56.

[1] See definition of international ampere in Appendix.

The silver nitrate solution is poured into the stock bottle through a filter paper to save any crystals of silver that may break off. A 5% silver nitrate solution gives the best deposit for currents of less than 0.5 ampere, but any solution up to 30% may be used. The area of the cathode should be between 50 and 500 square centimeters per ampere.

*To clean the silver cathode:*

If the cathode cup contains deposited silver crystals, it is usually clean, and needs drying only. If the crystals are not clean, the cup must be polished inside and then cleaned as directed:

1. Polish with clean cloth, water and fine sand
2. Wash to remove sand.
3. Wash with soap and water, and then with water alone.
4. Wash with a warm solution of potassium cyanide (1%). Wash in distilled water.
5. Dry in hot air bath at a temperature between 100° and 120° C., and leave to cool in a desiccator before weighing.

*To wash cathode after deposit:*

1. Rinse four times with hot distilled water pouring the water each time through a filter to save any loose crystals whose weight must be added to that of the cathode.
2. Fill with hot distilled water and let stand about thirty minutes. The final washing should give no reaction with HCl.
4. Dry in air bath as before and cool, before weighing. If a large deposit of crystals is present, keep cathode in air bath at least twenty minutes.

NOTE.—Use great care not to touch the inner surface of the cathode cup with the fingers during and after washing. It is best to place a large resistance in the circuit so that any variations in

the resistance of the voltameter can cause but little change in the strength of the current.

Copper, water and zinc voltameters are sometimes used but are not as accurate for small currents as the silver voltameter.

REFERENCES.—Stewart & Gee, II. p. 253. Henderson, p. 151. K. E. Guthe, Bulletin Bureau of Standards, Vol. 1, pp. 21 and 349; 1904–5.

## EXPERIMENT 58

### THE RELATION BETWEEN THE CURRENT AND ITS DEFLECTION ON A MOVING COIL GALVANOMETER

**Apparatus.**—For most electrical measurements the tangent galvanometer is not sufficiently sensitive. There are two types of sensitive galvanometers; in one, the coil is fixed, and the magnet is movable; in the other the magnet is fixed, and the coil is movable. The first type is illustrated in Fig. 57. This differs from the tangent galvanometer in having a coil of small diameter and a comparatively large number of turns of wire. The sensitiveness of the instrument is varied by sliding the coil nearer to or farther from the magnetic needle. The second type is known as the moving-coil or d'Arsonval galvanometer and is illustrated in Fig. 58 and by the ammeter, Fig. 54. The permanent

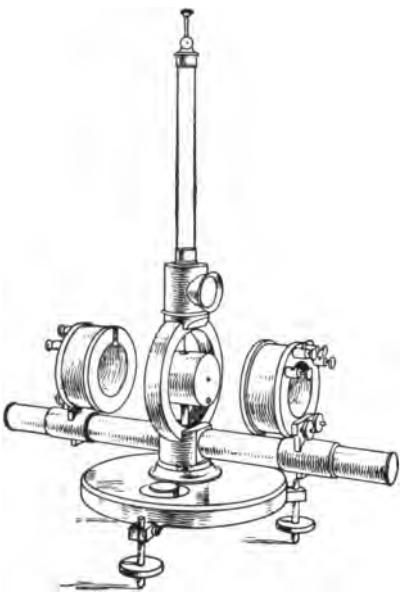


FIG. 57.

magnet serves as the frame of the instrument, and the coil is suspended in the magnetic field between the poles of this magnet. The sensitiveness is varied by means of an external shunt which allows only a part of the current to flow through the galvanometer coil. This type of galvanometer is not appreciably affected by external magnetic disturbances, and, therefore, is the type now most commonly employed.



FIG. 58.

**Method and Manipulation.**—A milliammeter *A* and a shunted moving-coil galvanometer *G* are connected in series with a resistance box *R* and a battery *B*. The galvanometer is also provided with a short-circuiting key *K*.

The sensitiveness of the galvanometer is adjusted until both instruments give about a full scale deflection. After adjustment, the sensitiveness of the galvanometer

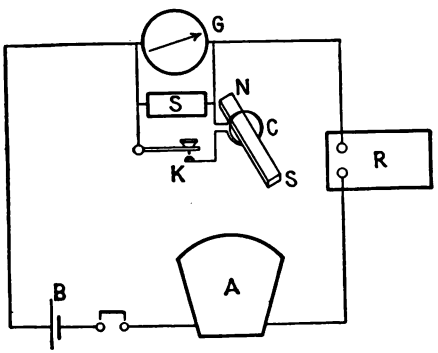



FIG. 59.

must not be changed during the experiment. Deflections are taken with about ten different currents, decreasing in value uniformly from the highest to the lowest.

The deflections of the galvanometer should preferably be to the right as viewed through the reading telescope. The magnitude of a deflection must be reckoned from the zero reading taken immediately after the deflection. If the galvanometer is not aperiodic, i.e., if on returning to the zero point the coil swings beyond it, the coil must be made aperiodic while it is returning, by short-circuiting the galvanometer through the key shown in Fig. 59. In case this does not produce a sufficient amount of damping, the motion of the galvanometer coil may be checked by induced currents produced by moving a magnet within a coil $C$ placed for that purpose in the short-circuiting branch. This operation is made necessary by the presence of magnetic impurities[1] within the coil which have their direction of magnetization changed as the coil passes into different parts of the magnetic field. The magnetic impurities affect[2] the sensitivity of the instrument and their large magnetic hysteresis is the cause of the change in the zero point. Even with this precaution, the zero point will usually change after successive deflections of increasing magnitude. In such cases it is desirable to take the largest deflections first. This effect is not observed in the less sensitive portable galvanometers and is often negligible in galvanometers when the coil is placed so that its axis passes accurately through the center of a uniform radial field.

The temperature coefficient[3] of a moving coil galvanometer for continuous currents is about 0.00018 when the upper suspension is of phosphor-bronze, and 0.00005 when it is of steel.

**Calculations.**—The strength of the current in the

[1] Physical Review, Vol. XXXII, p. 297, 1911.
[2] Physical Review, Vol. XXIII, p. 400, 1906.
[3] Physical Review, Vol, XXVIII, p. 277, 1909.

main circuit is given by the ammeter. This same current, or a fractional part of it, passes through the sensitive galvanometer and produces there the observed deflection. Plot a curve on coördinate paper, showing the relation between the current and the deflection produced on the sensitive galvanometer. The scale used in plotting must be large enough to show the relation to the limits of observational error. A record of the distance from the galvanometer mirror to the scale and a statement of the direction of the deflections should be made on the same coördinate paper.

NOTE.—In many electrical measurements it is only necessary to determine the ratio of the electric currents flowing through the galvanometer. The curve here obtained is used for determining this ratio. Usually the currents are proportional to the deflections for some distance so that for deflections within this range

$$\frac{i}{i'} = \frac{d}{d'}.$$

## EXPERIMENT 59

### TO MAKE AN AMMETER FROM A MOVING COIL GALVANOMETER

**Apparatus.**—Galvanometer; ammeter; battery; key; small wire resistance; resistance boxes; rheostat.

**Method and Manipulation.**—The galvanometer with a resistance box in its circuit is connected to the extremities of a small wire resistance $S$. This combination makes an ammeter, and, by adjusting the resistance $r$, a convenient whole number of divisions is made to represent a current given by some whole number of divisions on the ammeter $A$, Fig. 60.

Since the galvanometer deflections are usually not exactly proportional to the current, a *correction curve* or *calibration curve* must be made when accurate values are required from the instrument.

Compare the readings of the "galvanometer-ammeter" with the readings of the ammeter in the circuit at five or more points uniformly distributed along the scale.

Assign appropriate current values to the abscissas on coördinate paper, and, for the observed values of the current, plot the errors in the readings of the galvanometer-ammeter as ordinates. These errors should usually be plotted on a much larger scale than that employed for the abscissas. When the galvanometer-ammeter reads too high, the errors are plotted above the line of reference. This, however, is not a universal practice, and, for that reason, the proper notation must be made on the paper. The use of a *correction factor*, explained in Exp. 56, is less accurate than the use of a *calibration curve.*

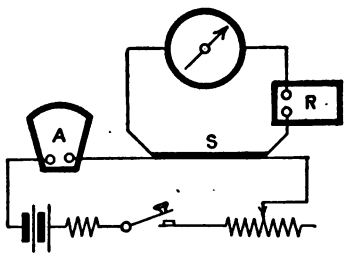


FIG. 60.

If a suspended coil galvanometer is used, the best arrangement of resistances is such as, with the aid of the the damping rectangle, makes the galvanometer just aperiodic.

## EXPERIMENT 60

### DETERMINATION OF THE DIFFERENCE OF POTENTIAL BETWEEN TWO POINTS ON A CONDUCTOR CARRYING A KNOWN ELECTRIC CURRENT—CALORIMETER METHOD

**Apparatus.**—Electric calormieter; ammeter; battery; rheostat; key; watch.

An electric calorimeter differs from an ordinary cal-

orimeter in having a resistance coil immersed in the liquid. It is used for measuring the heating effect of a current sent through this coil.

**Method.**—The difference of potential between two points in absolute units is measured by the work done in moving a unit quantity of electricity from one point to the other. All the work is transformed into heat within the conductor. The amount of this heat $H$, in calories, can be measured by means of the calorimeter. Whence from the definition, the difference of potential

$$E' = \frac{\text{work done}}{Q'} = \frac{JH}{I'T} = \frac{J(MS + M'S' + K)(t - t')}{I'T}$$
$$= \frac{10J(MS + M'S' + K)(t - t')}{IT},$$

where $J$ is the mechanical equivalent of heat.

As the specific heat of water varies with the temperature, the amount of work required to raise a gram of water 1° C. varies:

at 15° C. $J = 4.189 \times 10^7$ ergs,
20° C. $J = 4.179 \times 10^7$ ergs,
25° C. $J = 4.173 \times 10^7$ ergs,
30° C. $J = 4.171 \times 10^7$ ergs.

The practical unit of difference of potential is the *volt* and is equal to $10^8$ electromagnetic units. Thus

$$E = \frac{E'}{10^8}.$$

**Manipulation.**—The calorimeter with the stirrer, but without the coil and cover, is weighed. It is then filled to within about 2 cm. from the top with distilled water. The apparatus is connected as shown in Fig. 61, and, *being careful never to send a current through the calorimeter coil without having first immersed it in water*, adjust the

rheostat until the desired current, usually about 4 amperes, is produced. Disconnect the calorimeter, weigh it with the water and stirrer, and, then place it on ice to cool to about 5°C. below the temperature of the room. Wipe dry and connect it again into the circuit being careful that the key is first opened. The water is gently stirred during the whole experiment.

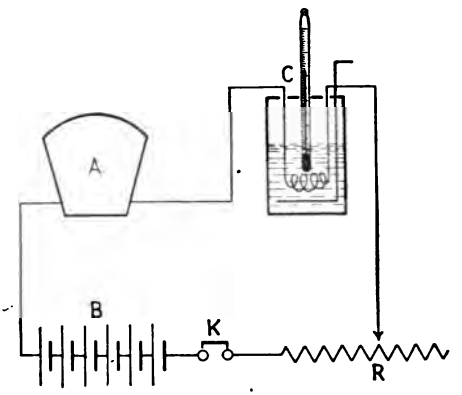



FIG. 61.

Before turning on the current, take readings of the temperature of the water at intervals of a minute for eight minutes. Then turn on the current, noting the time at that instant. Read the thermometer and ammeter every thirty seconds until the temperature has increased to a point as far above the temperature of the room as it was below at the instant the circuit was closed. Shut off the current, again noting the time at the instant the circuit was opened. Continue taking temperature readings at intervals of one minute for a period of eight minutes. Take outside temperature at intervals during the experiment.

Plot the readings taken, and from the curve determine

the rise in temperature by the construction explained in Exp. 37.

**Calculation.**—From the data obtained calculate the difference of potential in electromagnetic and in practical units, making allowance for the heat capacity of the coil immersed in the water.

## EXPERIMENT 61

### TO SHOW THAT BETWEEN ANY TWO POINTS ON A CONDUCTOR THE DIFFERENCE OF POTENTIAL VARIES DIRECTLY AS THE CURRENT—TO PROVE OHM'S LAW AND TO MEASURE A RESISTANCE BY THE CALORIMETER METHOD

**Apparatus.**—Same as in Exp. 60.

**Method.**—The difference of potential between two points on a conductor is determined for at least three different currents by the calorimeter method as in Exp. 60. Convenient values of the lowest and highest currents to be used are 3 and 5 amperes.

*A*. To show that the difference of potential and the current vary at the same rate, i.e., $\frac{E}{I}$ = a constant, plot on coördinate paper the values of $E$ and $I$ obtained in the experiment. The value of the constant depends on the length, thickness, and material of the conductor, and is called the *resistance* of that conductor. It is represented by the letter $R$.

The relation $\frac{E}{I} = R$, or $I = \frac{E}{R}$ is known as *Ohm's Law*.

*B*. Obtain the constant $R$ from the above plotted curve, thus determining the resistance of the conductor.

## EXPERIMENT 62

### TO TEST A VOLTMETER BY THE CALORIMETER METHOD

**Apparatus.**—Same as in Exp. 60, and a voltmeter.

**Manipulation.**—Attach the voltmeter to the extremities of the calorimeter coil used as in Exp. 60. Take the voltmeter readings and determine the fall of potential between the two points from calorimeter observations, or, having determined the resistance of the coil in Exp. 61, and having proven Ohm's law, the fall of potential may be calculated from the known resistance and the observed current by means of the equation $E = IR$.

Calculate the correction factor (see Exp. 56) of the voltmeter.

State why it is necessary for a voltmeter to have a high resistance.

## EXPERIMENT 63

### TO MAKE A VOLTMETER FROM A MOVING COIL GALVANOMETER

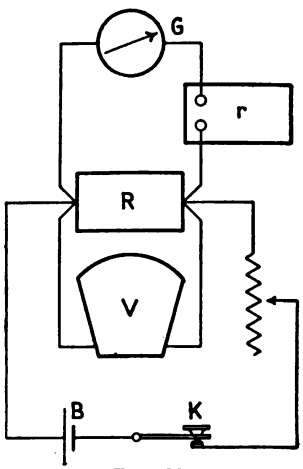



FIG. 62.

The voltmeter reads the difference of potential between its binding posts directly in volts. The moving part of the galvanometer corresponds to the moving coil of the voltmeter, and the only requirement necessary to change the galvanometer into a direct reading voltmeter is the introduction of the proper high resistance into its circuit.

**Apparatus.**—Galvanome-

ter; voltmeter; rheostat; two resistance boxes; battery; key.

**Manipulation.**—Connect the galvanometer and the voltmeter to the extremities of the same resistance as shown in Fig. 62. The high resistance $r$ in the galvanometer circuit is adjusted until some convenient whole number of large divisions on the scale represents one volt. If a box with a sufficiently high resistance is not available, a shunt may be placed next to the galvanometer to reduce its sensitiveness. This however should be avoided.

After the above adjustment has been made, vary the difference of potential between the extremities of $R$ by changing the resistance in the battery circuit, and, as in Exp. 59, obtain data and draw a calibration curve for the "galvanometer-voltmeter."

## EXPERIMENT 64

### ELECTROMOTIVE FORCE OF CELLS IN SERIES AND IN PARALLEL

**Apparatus.**—Two batteries; a voltmeter; the calibrated galvanometer-voltmeter of Exp. 63.

**Manipulation.**—Determine the electromotive force of each of the two cells, and then of the two in series and in parallel. Make the measurements with the galvanometer-voltmeter adjusted in the previous experiment and also with the voltmeter.

The electromotive forces may be compared with any unadjusted galvanometer having a sufficiently high resistance. The deflections are approximately proportional to the electromotive forces.

In any case, the galvanometer or the voltmeter measures only the difference of potential between its binding posts or between the binding posts to which it is con-

nected, by leads of negligible resistance. The fall of potential within the battery is not measured, and therefore the readings are too small by that amount. If the resistance of the voltmeter is large, this error is negligibly small.

## EXPERIMENT 65

### ELECTRO-CHEMICAL SERIES

The electromotive force produced by two substances in any given liquid depends on the nature of the substances. In a list arranged so that each substance is electro-positive to any one lower on the list, *the electromotive force between any two substances is equal to the sum of the electromotive forces between all of the intervening substances taken consecutively.*

**Apparatus.**—Dilute hydrochloric acid in a tumbler; clamps for attachment of substances to be tested; pieces of iron, zinc, copper, carbon and lead; calibrated high resistance galvanometer of Exp. 63; battery with marked electrodes.

**Manipulation.**—By noting the direction of the deflections produced by the battery and by the different pairs of substances when placed in dilute hydrochloric acid, make a list of these substances so that each is electropositive to the one below it. Then note the galvanometer deflections produced by each of the substances with the one next to it on the list; also by the first and last substance on the list. The readings are taken as quickly as possible to reduce errors due to polarization. The plates are washed before each repetition of the observations.

**Calculations.**—Correct the readings for the electromotive forces by use of correction curve of Exp. 63 and show that the e.m.f. of the first with the last

substance on the list is equal to the sum of the electromotive forces of all the intervening pairs of metals.

## EXPERIMENT 66

### RESISTANCE—SUBSTITUTION METHOD

**Apparatus.**—A sensitive galvanometer or a milliammeter; a battery; a switch; a rheostat; a resistance box; the resistance to be measured.

The resistance box contains a number of coils of wire of known resistance. These have been accurately measured by the manufacturer and properly marked. It is used for introducing a known resistance into an electric circuit, as is necessary in many electrical determinations. One of the uses is the comparison of an

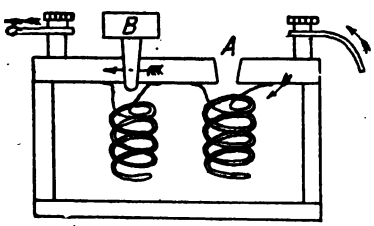



FIG. 63.

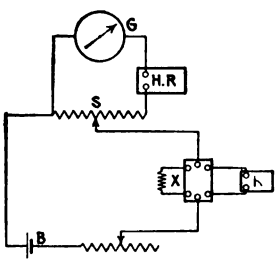



FIG. 64.

unknown resistance with the known resistances of the box, in a manner similar to the comparison of an unknown mass with the known masses of the balance weights. The coils are connected in series, as shown in Fig. 63. When a plug is removed, as shown at *A*, the whole current must pass through the coil of wire, and when the plug is in, as shown at *B*, practically the whole current passes through the plug. In order that the resistance of the plug may be negligible, it must be clean, and make a good contact.

When putting a plug into the box apply a *slight* press-

ure and turn to the right until a good contact is made. When removing it, turn either to the right or to the left. Use the box only when absolutely necessary and then for as short a time as possible. Heating of the coils by a current may permanently change their resistance, and, for this reason, too large a current must never be passed through them. (See Exp. 71.)

**Method and Manipulation.**—If a very sensitive galvanometer is employed it is usually connected to a shunt, *S*, with or without the additional resistance *HR*. When a milliammeter is used it is placed directly into the circuit. The remainder of the apparatus is connected as shown. With the switch turned to include the unknown resistance in the circuit adjust the current until a convenient large deflection is obtained. Then making sure that sufficient resistance has been introduced in the resistance box to protect the reading instruments, turn the switch to include the box in the circuit. Adjust the resistance until the galvanometer or milliammeter gives the same deflection as was obtained when the unknown resistance was in the circuit. The known resistance in the box must then equal the unknown resistance. If no set of resistances in the box gives exactly the same deflection the value of the unknown resistance is obtained by interpolation.

**Sources of Error.**—The resistance of the circuit may change during the experiment on account of variations in the battery, heating of the wire by the current, or changes in the room temperature. An error may also be introduced by the commutator if the resistances in its contacts or connections on the two sides are not exactly equal.

These errors are avoided by taking the final readings in rapid succession and by interchanging the positions of the compared resistances.

## EXPERIMENT 67

### MEASUREMENT OF A SMALL RESISTANCES-FALL OF POTENTIAL METHOD

**Apparatus.**—As shown in Fig. 65, except that a voltmeter may be used in place of G + $r$.

**Method.**—A current is passed through a known resistance $R$ and an unknown resistance $X$ placed in series. A high-resistance galvanometer, G + $r$, or a voltmeter is connnected successively to the extremities of these resistances by means of the switch C. The galvanometer, when connected to either $R$ or $X$, has a current flowing through it which is proportional to the fall of potential across these resistances, for in both cases the resistance of the galvanometer is the same. If the resistance of the galvanometer circuit is sufficiently high, its introduction as a shunt to either $R$ or $X$ does not appreciably change the fall of potential between the points to which it is attached. The ratio of the currents flowing through the galvanometer in the two cases is

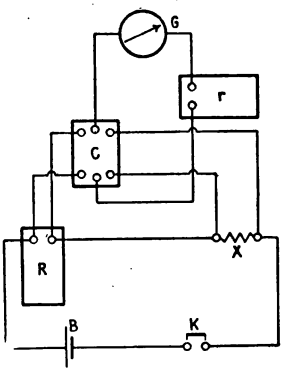


FIG. 65.

$$\frac{I}{I'} = \frac{E}{E'} = \frac{X}{R},$$

and

$$X = \frac{I}{I'} R = \frac{d}{d'} R.$$

when a voltmeter is used

$$X = \frac{E}{E'} R.$$

**Manipulation.**—Some suitable resistance is placed in $R$, and deflections are taken with the galvanometer connected in succession to the extremities of the two resistances. If, for the two cases, the deflections are not equal, a rough calculation of $X$ is made, and such a resistance is introduced in $R$ as is most nearly equal to this calculated value. A set of deflections is then taken for the final readings.

**Sources of Error.**—The experimental errors are reduced when the resistances, and therefore the deflections, are nearly equal. If $R$ and $X$ are not small compared with the resistance of the galvanometer, the introduction of the galvanometer changes appreciably the difference of potential between the ends of the resistance. But if $R$ and $X$, although not small, are made nearly equal, the introduction of the galvanometer lowers each difference of potential nearly the same fraction of itself so that the ratio of the differences of potential as they exist with the galvanometer connected is almost the same as their former ratio.

The resistence of wires parallel $= \frac{RG}{R+G}$. If $G$ is large compared with $R$, $\frac{RG}{R+G} = R$. In such a case the placing of $G$ in parallel with $R$ does not appreciably change the resistance of that part of the circuit between the ends of $R$ and therefore does not change the difference of potential. For this reason the introduction of the high resistance galvanometer does not change the difference of potential between the extremities of the resistances $R$ or $X$.

## EXPERIMENT 68

### FALL OF POTENTIAL ALONG A CIRCUIT

**Apparatus.**—Two low resistance batteries; three or four resistances of a few ohms each; a voltmeter.

**Method.**—The batteries and resistances are connected

in series. The fall of potential between the extremities of each of the resistances is measured in succession by means of the voltmeter and then the resistance between the extremities of all of them in series. The resistances are then disconnected from the batteries or the circuit is opened, and the voltage of the two batteries in series is measured. The experiment should prove that the sum of the differences of potential between the individual resistances is equal to the total fall of potential between the extremities of all of them in series and that the electromotive force of the batteries is also equal to this sum unless their resistance is appreciable.

## EXPERIMENT 69

### THE RESISTANCE OF A UNIFORM WIRE IS PROPORTIONAL TO ITS LENGTH

**Apparatus.**—High resistance galvanometer or a voltmeter; key; battery; rheostat; wire with sliding contact.

**Method and Manipulation.**—A constant current is passed through a long uniform high-resistance wire. A high-resistance galvanometer or a voltmeter is attached to one end of the wire and to a sliding contact which is first moved to a point 1, Fig. 66, and the reading is taken. The sliding contact is then moved to point 2, and a deflection is again taken. Continue in this manner in rapid succession until deflections for several equal steps have been taken. The small portions of the main current that flow through the galvanometer are to each other as the differences of

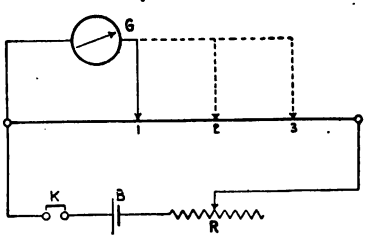


FIG. 66.

potential producing them. The ratio of the currents in the galvanometer is known from the deflections: $\frac{d}{d'} = \frac{i}{i'}$. Use curve of Exp. 58 if the deflections are large.

$$\frac{i}{i'} = \frac{e}{e'}.$$

If a voltmeter is used the differences of potential are read directly.

Plot a curve showing the relation between the differences of potential and the lengths of the wire included between the galvanometer terminals. This should show that

$$\frac{e}{e'} = \frac{l}{l'}.$$

But since the current, I, in the main circuit is constant, from Ohm's law.

$$I = \frac{e}{r} = \frac{e'}{r'}.$$

Then

$$\frac{e}{e'} = \frac{r}{r'}.$$

Therefore

$$\frac{r}{r'} = \frac{l}{l'}.$$

## EXPERIMENT 70

### TO DETERMINE A RESISTANCE BY MEANS OF A WHEATSTONE WIRE BRIDGE

**Apparatus.**—A Wheatstone wire bridge and accessories; the resistance to be measured.

**Method.**—The connections are made so that the known resistance $R$, and the resistance to be measured $X$, form two arms of the Wheatstone bridge, while the uniform wire $LL'$ forms the other two arms as shown in Fig. 67. The slide $S$ is moved along the wire until

the potential at $S$ is the same as that at $P$ so that no current flows through the galvanometer. The bridge is then said to be "balanced," and the fall of potential

$$e = e_1,$$

and

$$e' = e_1'.$$

But

$$\frac{R}{X} = \frac{e}{e'} = \frac{e_1}{e_1'} = \frac{L}{L'},$$

$$\therefore \quad X = \frac{L'}{L} R.$$

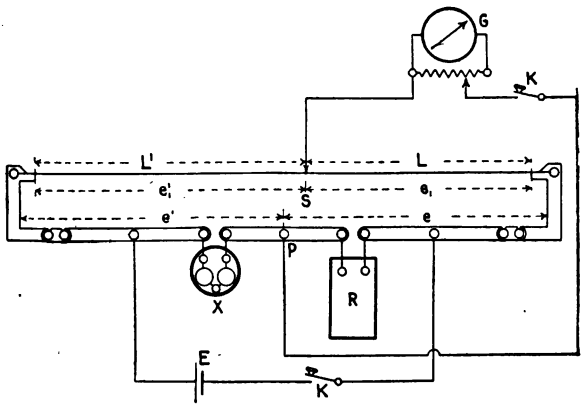



FIG. 67.

To avoid an error due to the connecting wires and to the contracts at the ends of the bridge wire, a balance of the bridge may be obtained with the galvanometer connected successively to the three places indicated in Fig. 68. It may be necessary to introduce small resistances $Y$ and $Z$ to bring the points $A$ and $C$ a short distance from the extremities of the bridge wire. $L$ and $L'$ are now the distances $BC$ and $AB$.[1]

[1] For high accuracy the bridge wire must be calibrated. See Carhart and Patterson, p. 78.

**Manipulation.**—The current is allowed to flow through the circuit only while an observation is being taken. The main circuit is closed an instant sooner than the galvanometer circuit to avoid a deflection by the instantaneous currents that may be induced in any part of the circuit at the time of closing. The galvanometer is first greatly decreased in sensitiveness by means of a shunt, and after a balance is roughly obtained the sensitiveness is increased again until a movement of a milli-

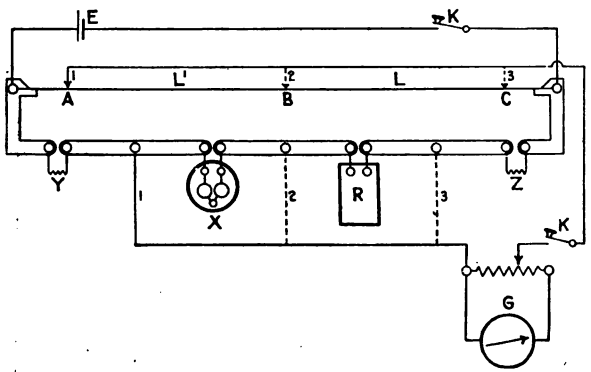



FIG. 68.

meter of the slide on the bridge wire causes a deflection of at least a millimeter on the galvanometer scale.

If a portable galvanometer is employed the shunt may have to be removed entirely for the final adjustment, or, the galvanometer may be used without a shunt, the sensitiveness being varied by means of a rheostat or a resistance box in the battery circuit.

The slide is pressed against the guide holding the scale, and when a final balance is obtained a reading is taken opposite the point of contact.

An approximate value of $X$ is obtained by a preliminary trial, and then the resistance in the box $R$ is

made nearly equal to this, so that in the final observations $L$ will be nearly equal to $L'$. Make another adjustment with $X$ and $R$ interchanged and average the result with that obtained by the first arrangement.

Determine the resistance of a coil of wire by the two methods of Fig. 67 and Fig. 68.

## EXPERIMENT 71

### TO DETERMINE A RESISTANCE BY MEANS OF THE WHEATSTONE BOX BRIDGE

The Wheatstone box bridge is a resistance box with keys and binding posts so placed as to make a convenient arrangement for use as a Wheatstone bridge. A diagram of one form of the bridge is shown in Fig. 69. The two sets of coils, $A$ and $B$, are called the "ratio arms" or

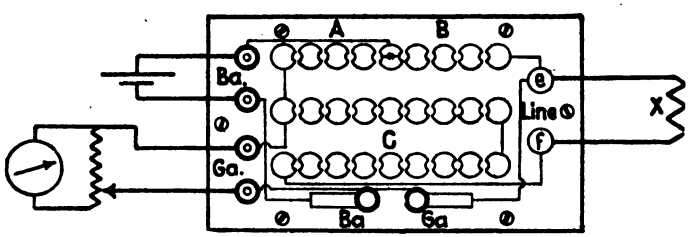



FIG. 69.

"bridge-arms," while the coils of the set $C$ are usually designated as the "rheostat" coils. The resistance to be determined is connected to the binding posts $e$ and $f$. A balance of the bridge is obtained in the same manner as in Exp. 70, except that instead of changing the ratio of $A$ to $B$ the resistance in $C$ is varied.

Since

$$\frac{A}{B} = \frac{C}{X},$$

$$X = \frac{B}{A} C$$

If the value of the resistance to be measured is known approximately, the ratio of the resistance in the ratio arms is made such as to require the highest possible resistance, in the rheostat arm to produce a balance of the bridge, but if it is not known, its approximate value may first be determined by using a ratio of unity, preferably 100 : 100. When the final balance is obtained, the resistance in the rheostat branch should be at least 1000 ohms, whenever possible, so that if the adjustment is made to within the nearest ohm, the resistance is measured to within 0.1%, the usual reliable limit of accuracy of an ordinary resistance box.

It is preferable to use only one coil in each ratio arm, and to avoid as far as possible the use of the one ohm ratio coils.

The galvanometer circuit should be closed first to test for thermo-electric currents in the circuit. If any deflection of the galvanometer is observed, the bridge must be balanced to this deflected position and not to the null point, after introducing as high a resistance into the battery circuit as the sensitiveness of the galvanometer permits.

If the circuit contains self-induction, first the battery circuit and then the galvanometer circuit must be closed. This is the usual procedure after the test for thermo-electric currents is made.

The circuit should be closed for a short time only, to avoid heating the coils and changing their resistance. The current-carrying capacity of the coils in an ordinary resistance box is about 0.25 watt. When necessary, 0.5 watt may be used for a short time only.

In the measurement of small resistances, the resistance of the connecting wires may not be negligible. It must then be determined separately.

Measure the resistance of the coils used in Exps. 67

and 70, and compare results; also measure the resistance of two coils separately, in series, and in parallel. Compare the last two results with values obtained by calculation from the determined resistances of the individual coils.

## EXPERIMENT 72

### THE TEMPERATURE COEFFICIENT OF COPPER WIRE

**Method.**—Determine the resistance of a coil of wire at the temperature of melting ice, and then at the temperature of steam.

Let $R_0$ = the resistance at 0° C., and $R'$ at the temperature $t$.

Then

$$R' = R_0(1 + \alpha t), \text{ and } \alpha = \frac{R' - R_0}{R_0 t}.$$

**Apparatus.**—A coil of well insulated copper wire immersed in oil within a metal tube; accessory apparatus for placing tube into ice and into steam.

**Manipulation.**—The coil must be surrounded by steam or ice for at least fifteen minutes before observations are taken to insure the temperature of the wire being the same as that of the ice or steam. The temperature has reached its final value when the resistance does not change with time. The temperature of melting ice is known and that of the steam is preferably determined from a barometric reading.

## EXPERIMENT 73

### MEASUREMENT OF VARIOUS RESISTANCES

Determine with the Wheatstone box bridge:

1. The resistance of a coil of copper wire at 20° C.
2. The resistance per foot of a copper wire of known

gauge, at 20° C. Calculate from the result the specific resistance of copper.

3. The resistance of a high resistance coil at 20° C.
4. The resistance of a carbon megohm.
5. The resistance of the human body with dry and then with moist contacts. For moist contact insert each thumb into a tube containing salt water.
6. A resistance containing a small electromotive force.
7. A resistance with self-inductance.

## EXPERIMENT 74

### MEASUREMENT OF RESISTANCE BY THE AMMETER-VOLTMETER METHOD

Resistances that are not larger than a few ohms may be determined by measuring the current in the circuit of which they are a part and the difference of potential between their binding posts.

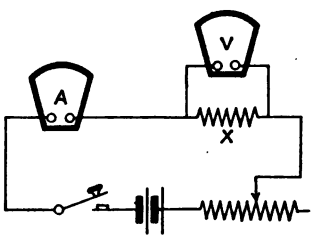



FIG. 70.

Then from Ohm's law the resistance

$$X = \frac{E}{I}.$$

Measure the resistance of the coil employed in Exp. 67.

## EXPERIMENT 75

### MEASUREMENT OF HIGH RESISTANCE—DIRECT DEFLECTION METHOD

**Apparatus.**—Galvanometer; megohm; high unknown resistance; two cells.

**Method and Manipulation.**—The Wheatstone bridge is not suitable for measuring resistances of many million

ohms. The direct deflection method is employed when the resistances to be measured are not too high. The galvanometer is first connected in series with a megohm (million ohms) and the deflection is observed. It is then connected with the unknown high resistance, and a battery of such electromotive force is used as will produce nearly the same deflection given in the previous case. If the same cell is not used, both electromotive forces must be measured. The resistance of the battery being assumed negligible,

$$i = \frac{E}{R},$$

$$i' = \frac{E'}{X},$$

$$\frac{i}{i'} = \frac{E}{E'} \cdot \frac{X}{R},$$

but

$$\frac{i}{i'} = \frac{d}{d'},$$

therefore,

$$\frac{d}{d'} = \frac{E}{E'} \cdot \frac{X}{R},$$

and

$$X = \frac{d}{d'} \cdot \frac{E'}{E} \cdot R.$$

## EXPERIMENT 76

### SPECIFIC RESISTANCE OF AN ELECTROLYTE

**Apparatus.**—A small induction coil from which the condenser has been removed; battery; sensitive telephone receiver; Wheatstone box bridge; electrolyte in vessel with suitable electrodes.

On account of polarization, the resistance of an electrolyte cannot be measured by any direct current method with accuracy. The polarization is avoided by using

a rapidly alternating current from the secondary of an induction coil or a transformer in place of the steady current from a battery. The time that the current continues in one direction is so short that no appreciable accumulation can form at the electrodes to increase the resistance or to produce an opposing difference of potential. The telephone is a very delicate detector of

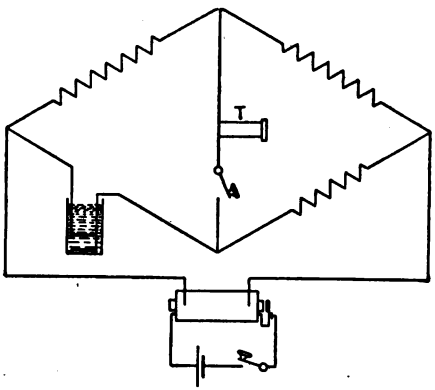



FIG. 71.

alternating currents, and must be substituted for the galvanometer. An alternating current galvanometer may also be employed.

**Manipulation.**—Measure the resistance of an electrolyte of known density at a known temperature in a tube of known diameter, using platinum plate electrodes. Change the separation of the electrodes by a known amount, and again measure the resistance. The difference in the resistances gives the resistance of a known column of the electrolyte. This eliminates uncertainty as to any remaining polarization at the electrodes and as to the exact location of the ends of the column.

From the resistance and the dimensions of the column, calculate the specific resistance.

## EXPERIMENT 77

### RESISTANCE OF A CIRCUIT AND THE ELECTROMOTIVE FORCE OF A BATTERY—OHM'S METHOD

**Apparatus and Method.**—Connect a battery in series with an ammeter, a rheostat and a resistance box. Measure the current with no resistance in the box, and then with a resistance $R$. Let $r$ be the resistance of the circuit. Then for the first case

$$I = \frac{E}{r},$$

and for the second case

$$I' = \frac{E}{r + R},$$

whence

$$\frac{I}{I'} = \frac{r + R}{r},$$

and

$$r = \frac{I'R}{I - I'}.$$

The resistance of the circuit then is known for each of the two known currents and $E = Ir = I'(r + R)$.

## EXPERIMENT 78

### ELECTROMOTIVE FORCE OF A BATTERY—POGGENDORFF'S METHOD

**Apparatus.**—As shown in Fig. 72.

**Method.**—Connect the cell $E$, whose e.m.f. is to be determined, as indicated in Fig. 72. The battery $B$ must have a greater e.m.f. than $E$. A definite resistance is placed in $R$, and the resistances $r$ and $S$ are varied until the sensitive galvanometer shows no deflection when the key $K$ is closed. The difference of potential $E'$ between the ends of the resistance $R$, produced by the

battery $B$, must then be equal to the e.m.f. of the cell $E$, because the two acting in opposite directions through the galvanometer circuit produce no current. Then

$$E = E' = IR.$$

The current $I$ is measured by means of an ammeter and $R$ is known.

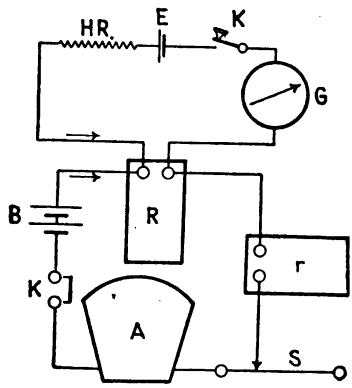


FIG. 72.

The proper value of the resistance to be placed in $R$ is calculated from a knowledge of the approximate value of $E$ and of a current that will give nearly a full scale deflection on the ammeter employed. This will insure a large deflection on the ammeter when the balance is obtained. The proper resistance is placed in $R$ and the approximately correct resistance in $r$ before the circuits are closed.

## EXPERIMENT 79

### ELECTROMOTIVE FORCE OF A BATTERY—POTENTIOMETER METHOD

**Apparatus.**—A sensitive galvanometer; a battery whose electromotive force is to be determined; a standard cadmium cell; a battery of constant e.m.f., preferably a storage cell; resistance boxes, etc. A crystal type of the cadmium cell is shown in Fig. 73. Its e.m.f. is 1.0183 volts and its temperature coefficient for most purposes is negligible. (See Appendix, Table 32.)

A high resistance must be placed in the circuit of the

standard cell to protect it from excessive currents. The electromotive force of a standard cell originally was determined by Poggendorff's method, and since, with proper usage, it remains constant, it is used as a standard for the determination by comparison of unknown electromotive forces.

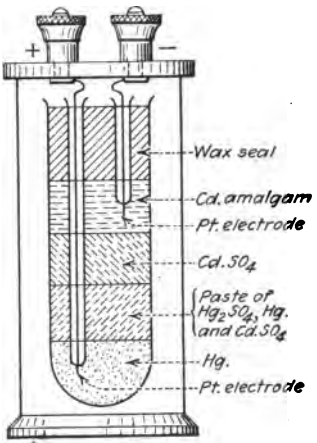



FIG. 73.

The standard cell is *never used* as a source for the production of electric currents. Its electromotive force *must not* be tested with a voltmeter. No current is supposed to be taken from it.

**Method.**—The potentiometer is an apparatus for comparing falls of potential by a modification of Poggendorff's method. For comparing electromotive forces of batteries, the apparatus is arranged as shown in Fig. 74. $B$ is a battery whose e.m.f. is greater than the e.m.f. to be determined. $E_s$ is a standard cell whose e.m.f. is known. The resistance of the galvanometer circuit should be at least 10,000 ohms to protect the standard cell. The cells are connected in opposition; a large resist-

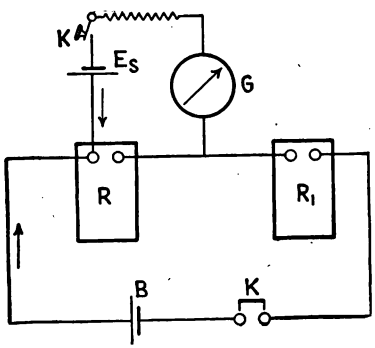



FIG. 74.

ance is placed in $R$, and then $R_1$ is adjusted until there is no deflection on the galvanometer. The fall of potential in the main circuit between the ends of the box must then equal the e.m.f. of $E_s$, and is equal to $\frac{E_s}{R'}$ per ohm, $R'$ being the resistance in the box $R$. The standard cell is then replaced by the battery whose e.m.f. is to be determined. The resistances in $R$ and $R_1$ are then adjusted, their sum being maintained constant, until the fall of potential between the extremities of $R$ is equal to the e.m.f. of the cell. The fall of potential along the line of the main circuit then is $\frac{E_x}{R''}$ per ohm, where $R''$ is the resistance in $R$. But since $R + R_1$ is constant, the current in the circuit is constant and therefore the fall of potential per ohm is constant, and

$$\frac{E_s}{R'} = \frac{E_x}{R''},$$

from which

$$E_x = \frac{R''}{R'} E_s.$$

**Manipulation.**—If the resistance $R'$ originally placed in $R$ is numerically 1000 times as large as the e.m.f. of the standard cell, the e.m.f. of the battery $E_x = \frac{R''}{1000}$. This is usually done as it simplifies the computations.

A commercial form of the potentiometer is shown in Fig. 80. This accomplishes somewhat more conveniently the same object as the resistance boxes of Fig. 74.

REFERENCES.—Cadmium cell, Phil. Mag. V, 48, 1899, p. 152; Bulletin of the Bureau of Standards, 1907, Vol. IV, p. 1.

## EXPERIMENT 80

### TO CALIBRATE A VOLTMETER BY THE POTENTIOMETER METHOD

1. A simple arrangement of apparatus for calibrating points on a voltmeter from the 1.1 volt point upward is shown in Fig. 75.

**Method and Manipulation.**—The standard cell is placed at $E_s$, and the resistance $R$ is made numerically equal to 1000 times the e.m.f. of the standard cell. The resistance $R_1$ is made such that $R + R_1$ is numerically equal to 1000 times the voltage reading to be tested. For example, in testing the 1.500 volt point the resistance $R + R_1$ is made equal to 1500 ohms. A resistance is placed in $R_2$. The circuit of the battery $B$ is then closed, and the resistance $R_2$ is adjusted until the voltmeter indicates the voltage to be tested. The standard cell circuit is then closed for the *first time*. If no deflection is produced on the sensitive galvanometer, the potentiometer is balanced, and the fall of potential between $X$ and $Y$ equals the electromotive force of the standard cell. Then the fall of potential in $R$ is one volt for every 1000 ohms, and between $X$ and $Z = \frac{R + R_1}{1000}$. If $R + R_1 = 1500$ ohms, the fall of potential between

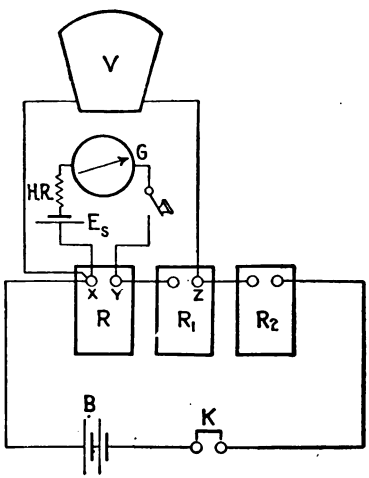



FIG. 75.

$X$ and $Z = \frac{1500}{1000} = 1.500$ volts. The current through the voltmeter is caused by the difference of potential between these points $X$ and $Z$, to which the voltmeter is connected. If the voltmeter indicates the determined value of the difference of potential, it reads correctly.

If a deflection is produced on the galvanometer when the standard cell circuit is closed, $R_2$ must be adjusted until there is no deflection. The difference of potential between $X$ and $Z$ will then be 1.500 volts as before, but the voltmeter may indicate some other number. The difference between the two is the error of the voltmeter reading at that point. To test whether the galvanometer has sufficient sensitiveness, change $R^2$ by an amount that will cause the voltmeter to change its reading by one-tenth of its smallest division. If the galvanometer then gives an appreciable deflection it is sufficiently sensitive.

The corrections for the other points on the voltmeter are determined in a similar manner by first placing the proper resistance in $R_1$ and then adjusting by means of $R_2$. If $R_2$ does not have a sufficient range of adjustment the resistance of the circuit can be changed by use of a sliding resistance. An adjustment to the nearest one-tenth of an ohm is usually sufficient.

Calibrate a 1.5 volt scale using the points 1.1, 1.2, 1.3, 1.4 and 1.5. Tabulate results, showing the percentage of error in each case. Use the sign + or − to show whether the correction is to be added to or subtracted from the observed reading. If the per cent. of error is the same in each case within the limits of observational error, this per cent. is called the *correction factor* of the voltmeter, and by its use readings can be corrected without referring to the calibration table. Draw a correction curve as in Exp. 59.

Is the fall of potential per ohm in $R_2$ the same as in $R$?

Should the voltmeter branch be provided with a key?

How does the strength of the current in the voltmeter compare with that in $R$?

Explain why it is possible to calibrate a voltmeter when it is connected to points between which the resistance is high.

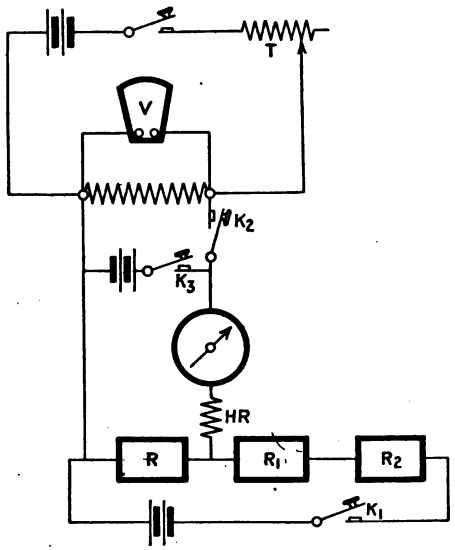



FIG. 76.

Why is it then that the voltmeter cannot be used to measure the voltage between points between which the resistance is high? Does it measure the voltage as it is after the voltmeter is connected?

What is an electrostatic voltmeter? What are its advantages?

II. An arrangement for calibrating any point on a voltmeter is shown in Fig. 76.

**Method and Manipulation.**—The standard cell $E_s$ is balanced as in the previous arrangement. The standard cell circuit is then opened and the key $K_2$ is closed. Keeping the value of $R + R_1$ constant as in Exp. 79, the fall of potential between the extremities of the resistance $S$ is obtained as though it were an electromotive force of a battery. The voltmeter being connected to the same points gives a reading of the same voltage, and from the difference the correction for the reading is obtained. The resistance $S$ need not be known. The voltmeter

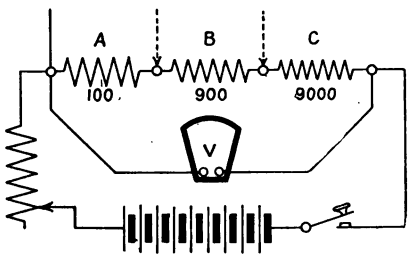



FIG. 77.

reading is varied by changing the current by means of the rheostat $T$. The values of the resistances to be placed in $R$ and $R_1$ are known approximately for any point to be calibrated on the voltmeter. These are properly transferred before any experimental adjustment is attempted.

Calibrate several points on the voltmeter from the 1.1 volt point to 0.1 volt. Add these corrections to the correction curve obtained for the higher values by the other arrangement. The correction for the 1.1 volt point should be the same by the two methods.

III. For calibrating higher than 15 volts it is usual to employ a volt box. This consists of three resistances, $A.B,C$, in series having values of 100, 900, and 9000 ohms

respectively. The current is passed through the three resistances and the voltmeter is connected to their extremities as shown in Fig. 77. If the fall of potential between the extremities of $A$ is measured by means of a potentiometer the voltage at the extremities of the box is 100 times that. If the fall of potential between the extremities of $A + B$ is measured the voltage is 10 times this value.

## EXPERIMENT 81

### TO CALIBRATE A MILLIAMMETER BY THE POTENTIOMETER METHOD

**Apparatus.**—The diagram shown in Fig. 78 is the same as that used in Exp. 78, except that $E_s$ is a standard cell.

**Manipulation.**—The potentiometer is balanced in the same manner as in Exp. 78. The difference of potential

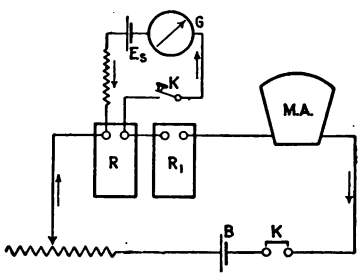



FIG. 78.

between the ends of the known resistance is equal to the e.m.f. of the standard cell and thus the value of the current in the circuit is $\frac{E_s}{R}$. For other values of the current $R$ is changed, and $R_1$ adjusted until the balance is again obtained. The results are tabulated, and the correction factor obtained as in Exp. 80.

As in all similar cases the value of $R$ is first calculated

for any particular value of the current. The electromotive force of the standard cell is fixed and therefore to balance it for various currents different values of $R$ must be employed. It is usually not possible to obtain the exact value of the current desired so that calibrations must be made for those values which the resistances in $R$ permit.

It is often just as convenient to calibrate a milliammeter by the methods described for calibrating an ammeter.

## EXPERIMENT 82

### TO CALIBRATE AN AMMETER BY THE POTENTIOMETER METHOD

**Apparatus.**—The apparatus may be arranged as shown in either Fig. 79 or Fig. 81.

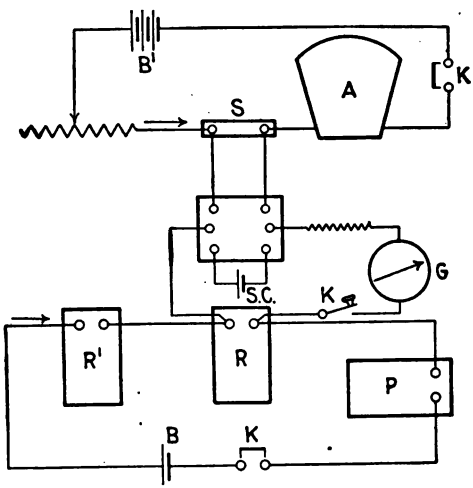



FIG. 79.

**Method and Manipulation.**—I. In Fig. 79 the current from the storage battery $B'$ is passed through the am-

meter to be calibrated, a rheostat, and a standard low resistance $S$ made specially for use with large currents. The fall of potential between the extremities of $S$ is determined by comparison with a standard cell using the potentiometer arrangement of Exp. 79. By means of a commutator, the standard cell $S.C.$ is first connected to the resistance $R$ 1000 times larger than the e.m.f. of the cell, and a balance is obtained. The commutator bar is then turned and the fall of potential $E$ at the extremities of $S$ is determined. The current through the ammeter is then equal to $\frac{E}{S}$.

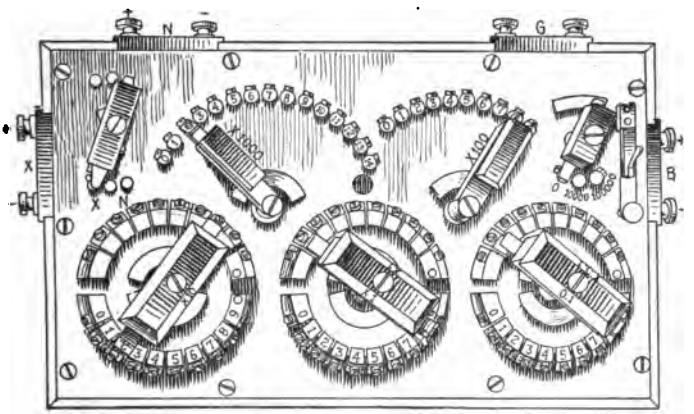



FIG. 80.

The approximate values of the resistances required to produce a balance are always first calculated from the approximately known value of the current.

Test at least five points on the ammeter properly distributed over the full range of the scale, and plot the correction curve.

Commercial instruments called "potentiometers," which are illustrated by the one shown in Fig. 80,

operate essentially in the above manner. All the keys and all the resistances except $P$ and $S$, are conveniently placed on and in one box. These instruments may be used to calibrate either voltmeters or ammeters or to measure any electromotive force within their range.

II. The other method is shown in Fig. 81. This has an advantage over the first method in that only one storage battery is required. The current in the main circuit is regulated until the ammeter gives the reading to be tested. The resistance in $R + R'$ must be at least 2000 times that in $S$. The current flowing through $R + R'$ is then a negligibly small portion of the whole current, and that flowing through $S$ may be assumed to be the whole current flowing through the ammeter.

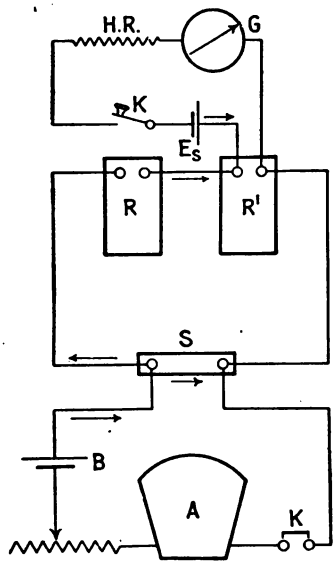


Fig. 81.

The value of the current flowing through the resistance $S$ is determined from the difference of potential $E$ between its extremities.

The standard cell is balanced against the difference of potential in the resistance $R'$, the approximate value of the resistance required for this being first calculated. The fall of potential in $R'$ included in the standard cell circuit is then equal to the electromotive force of the standard cell, and the fall of potential $E$, between the extremities of $R + R'$ and of $S$ can be calculated.

The current flowing through $S$ and the ammeter is

obtained from $I = \frac{E}{S}$. This is repeated for various values of the current.

If the current through $R$ is not a negligible portion of the whole, its value is calculated from the known difference of potential and resistance. Thus $i = \frac{E}{R + R'}$, and the total current through the ammeter is

$$I + i = \frac{E}{S} + \frac{E}{R + R'}.$$

The fall of potential between the extremities of $S$ must be larger than the electromotive force of the cell. This is the chief objection to this method especially when calibrating an ammeter through its whole range. The adjustment is often simplified by placing in $R_1$ a resistance 1000 times larger than the e.m.f. of the standard cell and then placing the proper resistance in $R$ to obtain a balance.

## EXPERIMENT 83

### CALIBRATION OF AN AMMETER—VOLTMETER METHOD

**Apparatus.**—Ammeter; voltmeter; standard resistance; battery; rheostat; key.

**Method.**—The current is passed through the ammeter in series with a rheostat and a standard resistance. A voltmeter that has been carefully calibrated is attached to the posts of the standard resistance. The current flowing through the ammeter is determined from the fall of potential between the ends of the known resistance $R$.

If the current flowing through the voltmenter is not a negligible portion of the whole current, its value is

determined from $i = \frac{E}{G}$, where $G$ is the resistance of the voltmeter. The total current flowing through the ammeter is $I + i = \frac{E}{R} + \frac{E}{G}$.

The errors in the readings are tabulated, and the correction curve is plotted as in the previous experiments.

**Precaution.**—The voltmeter should have such a sensibility that with the standard resistance used the deflections will be nearly equal to those of the ammeter.

NOTE.—The methods for calibrating ammeters are methods for measuring an electric current without the use of an ammeter. The current is measured in terms of difference of potential and resistance.

## EXPERIMENT 84

### THEORY OF THE BALLASTIC GALVANOMETER

A ballistic galvanometer is used for measuring the quantity of electricity that passes through the instrument during an instantaneous discharge such as is obtained from a condenser or from an induced current. Any galvanometer may be employed for this purpose and when so employed is called a "ballistic galvanometer." In place of a continuous deflection which an electric current produces, the quantity of electricity sets the moving part (magnet or coil) into motion and causes it to swing a definite distance depending on the quantity of discharged electricity. The distance of the swing, as represented on the reading scale, is called the "throw" of the galvanometer.

The galvanometer is more sensitive if the moving part is undamped, and the throws are more easily read if the period of vibration is long. If the coil is undamped, the throws are also more nearly proportional to the quantities of electricity; and if the period of vibration is long, the whole of the instantaneous current passes

through the coil before the moving part is appreciably deflected, a condition required by the theory of the instrument. For these reasons a ballistic galvanometer has its moving part undamped, and its period of vibration long.

If the moving part be damped so as to be aperiodic or nearly so, the constant of the galvanometer cannot be calculated, but can be determined by the indirect methods of Exps. 88 and 107, which are the most convenient methods to employ in any case.

The relation existing between the throw and the quantity of electricity is here derived for the moving-coil type of the galvanometer.

A. *Moment of the force tending to rotate the coil.*

When a current $I'$ flows through the coil in the direction indicated in Fig. 82, the coil, when viewed from the

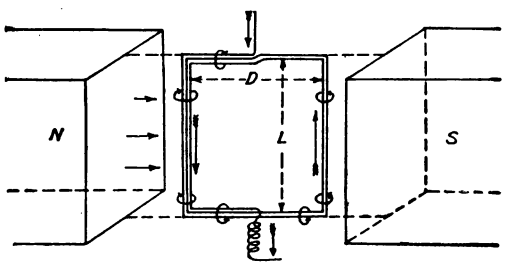



FIG. 82.

top, is deflected in a direction contrary to the hands of a watch. The moment of the force tending to produce rotation is all due to the current in the vertical parts of the coil.

If $F$ be the strength of the magnetic field and $n$ the number of turns on the coil, the force with which each vertical side of the coil is acted on, is $FLnI'$ dynes. When the plane of the coil is parallel to the magnetic

lines of force the moment of this couple is $FLnDI'$; and when it makes the angle $\phi$ with that direction, the moment is $FLnDI' \cos \phi$. (See Fig. 83.) In case of a radial field the plane of the coil, upon deflection, remains parallel to the lines of force and $\cos \phi = 1$.

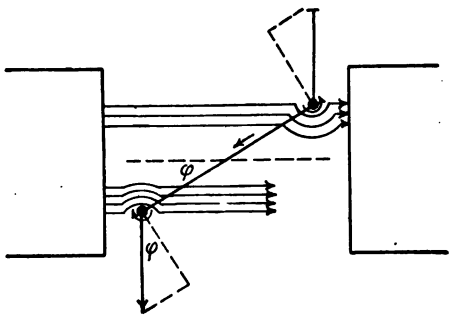



FIG. 83.

B. *The constant $K'$ of the galvanometer for continuous currents.*

The moment of the force producing rotation must equal the moment, $T\phi$, of the torsional couple, when the coil comes to rest. Thus $T\phi = FLnDI' \cos \phi$

$$I = 10I' = \frac{10T\phi}{FLnD \cos \phi} = K' \frac{\phi}{\cos \phi},$$

$$\therefore K' = I \frac{\cos \phi}{\phi},$$

and for a radial field $K' = \frac{I}{\phi}$ . . . . . (I)

$$\left[K' = \frac{10T}{FLnD} \cdot FLnD = \frac{10T}{K'}\right] \qquad \text{(II)}$$

C. *The constant $K$ giving the relation between the quantity of electricity and the throw of the coil.*

Let $I''$ be the mean value of the transient current.

During the passage of the current a force is exerted upon the coil, the moment of which is

$$fr = FlnDI'' = I_0\alpha.$$

Then

$$frt = FlnDI''t = I_0\alpha t$$

from which

$$FlnDQ' = I_0\omega$$

and

$$Q = 10Q' = \frac{10I_0\omega.}{FlnD}. \qquad \text{(III)}$$

This equation gives an expression for the quantity of electricity that passed through the galvanometer, but is not in terms of quantities that can be easily measured. It is now only necessary to substitute for these those quantities that can be measured with ease.

Substituting for $FlnD$ its value given in equation (II),

$$Q = \frac{10I_0\omega}{\frac{10T}{K'}} = \frac{I_0K'\omega}{T} \qquad \text{(IV)}$$

The kinetic energy acquired by the coil $= \frac{1}{2}I_0\omega^2$, which, neglecting damping, is all changed into potential energy when the coil reaches the end of the throw. The average torsional moment that the moving coil acts against in its motion to the end of the angle of throw $\theta$ is $\frac{0 + T\theta}{2} = \frac{1}{2}T\theta$. The work done by the moving coil in twisting the suspensions = the average torsional moment $\times$ the angular distance $= \frac{1}{2}T\theta \cdot \theta = \frac{1}{2}T\theta^2$. When all the kinetic energy has been converted into potential energy at the end of the throw,

$$\frac{1}{2}I_0\omega^2 = \frac{1}{2}T\theta^2,$$

$$\omega = \sqrt{\frac{T}{I_0}} \cdot \theta.$$

Substituting the value of $\omega$ in equation (IV),

$$Q = \frac{I_0 K'}{T}\sqrt{\frac{T}{I_0}} \cdot \theta = \sqrt{\frac{I_0}{T}} \cdot K'\theta \ldots\ldots\ldots (V)$$

The complete period of vibration of the galvanometer coil $t = 2\pi\sqrt{\frac{I_0}{T}}$, from which $\sqrt{\frac{I_0}{T}} = \frac{t}{2\pi}$. Substituting this value in equation (V),

$$Q = \frac{K't}{2\pi}\theta, \ldots\ldots\ldots\ldots\ldots (VI)$$

in which all the quantities can be easily measured. It has however been assumed that there is no damping in the motion of the coil.

D. *Damping correction.*—Equation (VI) is developed on the assumption that all the kinetic energy imparted to the coil by the electric discharge is changed into potential energy when the coil has reached the end of the throw. This, however, is not the case. Some of the energy is consumed in the production of induced currents, in overcoming the resistance of the air, and in the internal resistance of the suspending fiber. For this reason a correction must be applied to equation (VI).

The first throw of the coil is smaller than it would be if there were no damping. Any observed throw must be multiplied, therefore, by some factor that will increase its value to what it would have been if undamped.

While the coil is vibrating, the successive angles of vibration bear to each other a practically constant ratio. Thus

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \frac{\theta_{(n-1)}}{\theta_n} = \rho \text{ nearly.}$$
$$\theta_1 = \rho\theta_2 = \rho^2\theta_3 = \rho^3\theta_4 = \rho^{n-1}\theta_n.$$

For example, to find what the angle $\theta_3$ would have been had there been no damping in the last two vibrations,

$\theta_3$ is multiplied by $\rho^2$, i.e., by $\rho$ *with an exponent numerically equal to the number of intervening vibrations.*

When an actual throw is taken, the coil starts from the center of its natural arc of vibration and has moved, therefore, through but *one-half* of a single vibration. To find what the angle of throw would have been without damping, multiply the observed angle $\theta_1$ by $\rho$ with an exponent numerically equal to the number of intervening vibrations, i.e., $\frac{1}{2}$.

Then

$$\theta = \rho^{\frac{1}{2}}\theta_1,$$

Substituting this in equation (VI),

$$Q = \frac{K't\rho^{\frac{1}{2}}}{2\pi}\theta_1 = K\theta_1. \quad \text{.....(VII)}$$

$$\therefore \quad K = \frac{K't\sqrt{\rho}}{2\pi}. \quad \text{.....(VIII)}$$

For open circuit work $\rho^{\frac{1}{2}}$ is nearly constant, but for throws in closed circuit work the amount of damping is dependent on the resistance of the apparatus. The factor $\rho^{\frac{1}{2}}$ must be determined for each circuit employed. In moving coil galvanometers of low resistance, the amount of damping on closed circuit is usually so great as to make the instrument aperiodic. In such a case, the constant of the galvanometer cannot be determined by this method.

Magnetic impurities within the coil influence[1] somewhat the magnitude of the throw. Equation (VII) makes no correction for this effect which is usually not very large.

The temperature coefficient of this type of galvanometer for ballistic throws is about — 0.00017.

[1] Physical Review, Vol. XXIII, p. 400, 1906; Vol, XXXII, p. 297, 1911.

## EXPERIMENT 85

### DETERMINATION OF THE CONSTANT OF A BALLISTIC GALVANOMETER ON OPEN CIRCUIT

The constant of a ballistic galvanometer, $K = \frac{K't\sqrt{\rho}}{2\pi}$, is obtained by deterining $K'$, $t$, and $\sqrt{\rho}$.

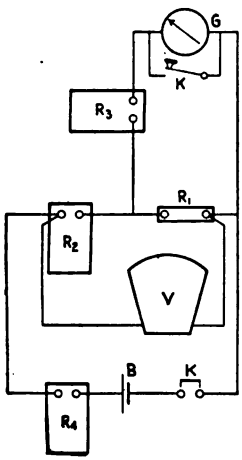


FIG. 84.

**Apparatus.**—As shown in Fig. 84.

**Method.**—A. *Determination of $K'$.*—The apparatus may be connected as shown in the figure. The resistance $R_1$ should not be larger than 5 ohms, but the resistance $R_2$ may have a value of several hundred ohms. The voltmeter $V$ gives the difference of potential between the extremities of $R_1 + R_2$, from which the difference of potential $e$ at the extremities of $R_1$ can be calculated. The resistance $R_3$ should have a value of about 10,000 ohms. If $G$ is the resistance of the galvanometer, the value of the current flowing through it is

$$I = \frac{e}{R_3 + G}.$$

If the angle of deflection produced by this current is $\phi$ radians, in galvanometers where the magnetic field is radial,

$$K' = \frac{I}{\phi}.$$

**Manipulation.**—The galvanometer coil is not aperiodic, but it should not be allowed to swing beyond the point of steady deflection or on returning to the zero point to pass beyond it. This is necessary on account of the

hysteresis of the magnetic impurities within the coil and is accomplished by tapping the short-circuiting key $K$ while the coil is in motion in either direction. The value of the angle $\phi$ is determined from the deflection, the zero reading taken after the deflection, and the distance of the mirror from the scale as explained in Exp. 48. A deflection is also taken in the opposite direction and the average value of $\phi$ is employed in the calculation of $K'$. It is desirable to have the magnitude of the deflection about 10 cm.

The magnetic field in most galvanometers is not uniform, and therefore the same current produces different deflections on the two sides of the zero point. For the purpose of determining the constant of a ballistic galvanometer, the value of $K'$ is desired for the field as it is at the zero point, and not as it may be in any deflected position of the coil. This can be obtained approximately from the average of the deflections on the two sides of the null point and by having these deflections as small as is consistant with the desired degree of observational accuracy.

A thermo-electric current may exist in the galvanometer branch of the apparatus. This affects the zero reading which must be taken with the thermo-electric current present, i.e., the galvanometer circuit must be left closed, while the short-circuiting key is open.

B. *Determination of t.*—Cause the coil to vibrate by deflecting it by means of a small current and then *opening* the circuit. Determine $t$ by the usual method. The passages of the zero mark on the scale are observed in the telescope. A wire may be hung over the zero mark, if necessary, to make the passages visible. A sufficiently large current may often be obtained for deflecting the coil by holding in each hand one of the wires leading to the galvanometer, the fingers being moistened.

C. *Determination of* $\sqrt{\rho}$.—Cause the coil to vibrate on open circuit and observe the scale readings at the extremities of the vibrations. When a circular scale is used,

$$\frac{d_1}{d_{n+1}} = \frac{\theta_1}{\theta_{n+1}} = \rho^n,$$

$$\sqrt{\rho} = \sqrt[2n]{\rho^n}.$$

The terms $d$ represent the observed lengths of any two arcs. The length $d$ is distance between the extreme ends of a vibration and not the distance from the zero point to one extreme.

In a moving coil galvanometer $\sqrt{\rho}$ varies somewhat with the amplitude of vibration. It is therefore desirable to obtain its value for nearly the same amplitude as that of the throw in any particular case. This is obtained by taking only a few vibrations immediately following a throw.

The $\sqrt{\rho}$ must be determined for each particular circuit when the galvanometer is employed on closed circuit. For open circuit work it has a definite value smaller than that on any closed circuit.

Obtain the value of $\sqrt{\rho}$ on open circuit for a throw of about 10 cm., using 5 vibrations for determining $\rho^n$.

D. Having determined $K'$, $t$, and $\sqrt{\rho}$, the value of the constant $K$ of the ballistic galvanometer for open circuit work, is calculated.

## EXPERIMENT 86

### QUANTITY OF ELECTRICITY IN A CHARGED CONDENSER

**Apparatus.**—Condenser; galvanometer; standard cell; high resistance; commutator; key.

**Method.**—The condenser is charged by connecting $A$ and $B$, Fig. 85 to a secondary standard cell in series with a high resistance.

It is then discharged by disconnecting $B$ from $A$ and immediately connecting to $D$. This is done by means of a key or a commutator.

$Q = K\theta_1$ coulombs,
$Q' = \frac{1}{10} Q$ electromagnetic units,
$Q'' = 3\text{x}10^{10} Q'$ electrostatic units.

Determine the quantity in each of the three units.

**Manipulation.**—To bring the galvanometer coil to rest, short-circuit the galvanometer by means of the key $K$. If on opening the short-circuit the coil does not remain at rest, a thermo-electric current existed during the short-circuit. If this is the case, estimate the position of the null point from the vibrations and again short-circuit the galvanometer. Then open the short-circuiting key and as the coil passes the known position of the null point, again short-circuit the galvanometer for an instant by tapping the key. With a little practice, the coil can quickly be brought to rest in this manner. If the vibrations are not completely removed when the throw is to be taken, the condenser is discharged when the coil is at either of its elongations; the throw, however, is computed from the null point and not from the elongation.

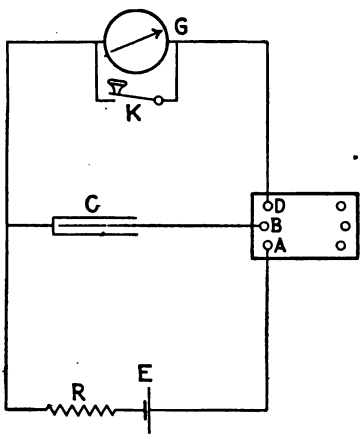


FIG. 85.

A high resistance galvanometer is not made aperiodic on short-circuit, and cannot be brought to rest

in this manner. If, however, a coil of wire is introduced into the short-circuiting branch, a proper motion of a magnet within this coil will produce the desired effect. When the galvanometer coil is at rest at the null point the short-circuiting key is opened.

In galvanometers that are aperiodic on closed circuit, the effect of the thermo-electric currents may also be eliminated by either one of the two following methods:

1. A thermo-electric couple is made from a piece of spring brass wire by annealing the central portion of it by heating. This is introduced into the short-circuiting branch of the galvanometer, and the coil is brought to the null point by warming with the fingers the proper part of the couple.

2. A small part of the galvanometer circuit is made a part of a secondary circuit containing a large resistance. The fall of potential in this part is made to oppose the thermo-electromotive force in the galvanometer and is adjusted by altering the large resistance until it is equal to it.

## EXPERIMENT 87

### CAPACITY OF MICA CONDENSER—ABSOLUTE METHOD

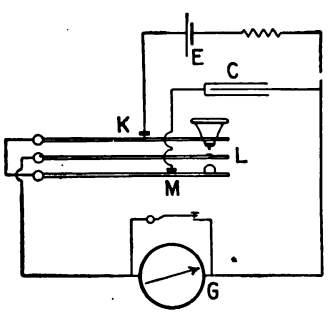



FIG. 86.

**Apparatus.**—The condenser, galvanometer, a special discharge key, and a secondary standard cell, are connected as shown either in Fig. 86 or Fig. 86*A*.

**Method and Manipulation.**—The object to be accomplished in either cases to charge the condenser by

means of a standard cell and then by means of one tap on the key or by the reversal of the commutator rocker, to open the battery circuit, connect the condenser to the galvanometer, and again open the circuit.

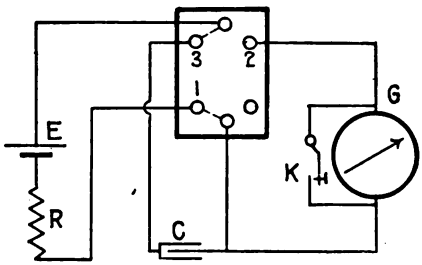



FIG. 86A.

In Fig. 86 a key with three metallic springs is employed. In the position shown, the condenser $C$ is being charged by means of the cell $E$. When the key is depressed, the spring $K$ opens the cell circuit, then the

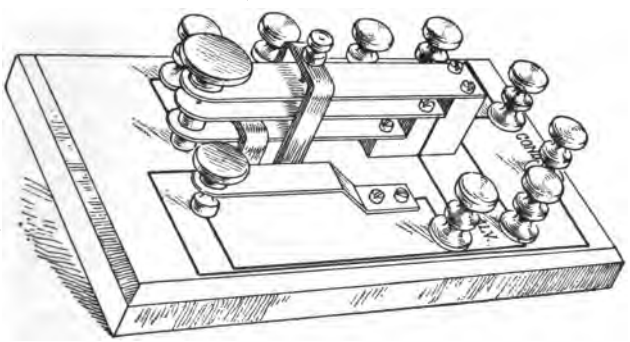


FIG. 87.

spring $K$ makes a metallic contact with spring $L$, and finally the spring $M$ opens the condenser circuit. This accomplishes in quick succession the above mentioned results. The key must be kept down until the full

throw is obtained. Fig. 87 shows a form of the key combined with a key for short-circuiting the galvanometer.

In Fig. 86*A* an ordinary Pohl commutator without cross-bars is employed. The height of the mercury in the cups is adjusted so that when the rocker is turned from the charging position, shown in the figure, the cell circuit is opened first at cup 1. The rocker then makes a contact in cup 2 before leaving cup 3, discharging the condenser during the time the two cups are in connection. The period of discharge can be varied by changing the height of the mercury in the cup 3.

The capacity determined in this manner is temporarily called the "free charge" or instantaneous capacity to distinguish it from the "ordinary" capacity value as obtained when the condenser remains in circuit with the galvanometer during the whole period of its throw.

When a condenser discharges, the free charge follows the law developed in Exp. 94. In ordinary circuits the time required to discharge the whole measurable part of the electricity is less than .01 second. The absorbed charge, however, continues to flow for an indefinite period. In the "free charge" capacity value[1] only a negligible portion of the absorbed charge is included, while in the "ordinary" capacity value the absorbed charge plays a considerable part, the magnitude of which depends on the period of the galvanometer. The latter is a variable quantity and has but little value.

Determine both the "free charge" and the "ordinary" capacity values of the condenser.

$$Q = K\theta_1$$

$$C = \frac{Q}{E} \text{ farads} = \frac{10^6 Q}{E} \text{ microfarads.}$$

[1] Physical Review, Vol. xxii, p. 65, 1906; Vol. xxvii, p. 65, 1908.

## EXPERIMENT 88

### CONSTANT AND FIGURE OF MERIT OF A BALLISTIC GALVANOMETER ON OPEN CIRCUIT—STANDARD CONDENSER METHOD

**Apparatus.**—As in Fig. 86, Exp. 87.

**Method and Manipulation.**—Discharge the free charge of the condenser of known capacity through the galvanometer as in Exp. 87.

$$Q = CE = K\theta = Fd.$$

$$K = \frac{CE}{\theta}; \quad F = \frac{CE}{d}.$$

$F$ = figure of merit, i.e., the quantity of electricity that produces a throw represented by one centimeter on the scale; $d$ = throw in centimeters.

This is an indirect method, and is the one most commonly employed when the galvanometer is to be used on open circuit. The *figure of merit* replaces the *constant* especially when a circular reading scale is used. The millimeter is employed by some in place of the centimeter as the unit of throw, but the centimeter will be used in this text. A new determination of the value of the figure of merit should be made before or after each experiment.

The term *figure of merit* is also employed in another sense.

$$C = \frac{Q}{E} = \frac{Fd}{E} \text{ farads} = \frac{10^6 Fd}{E} \text{ microfarads.}$$

But $\frac{10^6 F}{E}$ is a constant when the same electromotive force is employed. Let $f$ represent this constant. Then

$$C = fd.$$

It is seen that $f$ is a figure of merit which when multiplied by the throw gives the capacity in microfarads.

## EXPERIMENT 89

### CAPACITY OF A LEYDEN JAR

**Apparatus.**—As in Fig. 86, Exp. 87; Leyden jars.

**Method and Manipulation.**—Charge several similar Leyden jars, placed in parallel, with a known electromotive force of about 100 volts, and immediately discharge through the ballistic galvanometer.

$$C = \frac{Q}{E} = \frac{Fd}{E} \text{ farads} = \frac{10^6 Fd}{E} \text{ microfarads.}$$

The capacity of one jar

$$c = \frac{C}{N}.$$

The capacity in electrostatic units (centimeters)

$$= \frac{9 \times 10^{11} Fd}{E}.$$

Determine the capacity in microfarads and in electrostatic units.

Place a resistance of at least 10,000 ohms in series with the battery to protect the galvanometer against accident.

## EXPERIMENT 90

### THE AYRTON SHUNT

The Aryton shunt is constructed in such a manner that any galvanometer to which it is connected whatever may be its resistance, receives the same fraction of the whole current when the switch is on the same contact pin. This makes the shunt universal. It consists of a series of resistances represented

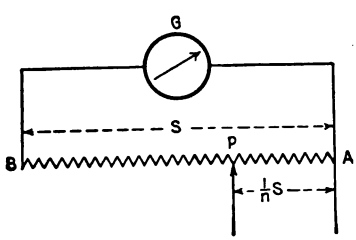



FIG. 88.

by $S$ in Fig. 88. The galvanometer is connected to the extremities of this series. The source of electricity is connected to $A$ and $P$, the latter being a switch which can be moved to include different fractions of the whole resistance $S$. Let $i_1$ be the current flowing through the galvanometer, $I$ the whole current, and $G$ the resistance of the galvanometer. Then if $P$ is connected to $B$ so as to include the whole of the resistance $S$ between $A$ and $P$.

$$\frac{i_1}{I - i_1} = \frac{S}{G}$$

from which

$$i_1 = \frac{S}{G + S} I \ldots\ldots\ldots\ldots\ldots \text{(I)}$$

When the switch includes $\frac{1}{n}$th of the whole resistance, between $A$ and $B$,

$$\frac{i_1'}{I - i_1'} = \frac{\frac{1}{n} S}{G + \left(S - \frac{1}{n} S\right)},$$

from which

$$i_1' = \frac{\frac{1}{n} S}{G + S} I = \frac{1}{n} \frac{S}{G + S} I \ldots \text{(II)}$$

From equations (I) and (II)

$$\frac{i_1}{i_1'} = \frac{1}{\frac{1}{n}},$$

when the whole current is the same in both cases. The ratio of the two currents is independent of the resistance of the galvanometer, and depends only on the ratio the whole resistance $S$ bears to the fraction of the resistance included between $A$ and $P$.

The resistances of the shunt have usually the ratios 1 : 9 : 90 : 900, so the resistances that may be included between $A$ and $P$ bear to each other the ratio of 1 : 10 : 100 : 1000. The corresponding ratios of the currents flowing through the galvanometer are 0.001 : 0.01 : 0.1 : 1. These latter ratios are usually stamped on the shunt as shown in Fig. 88A. When the switch is on the pin marked 0, the source of electricity is short-circuited, and when it is on the pin marked *Inf.*, the circuit is open.

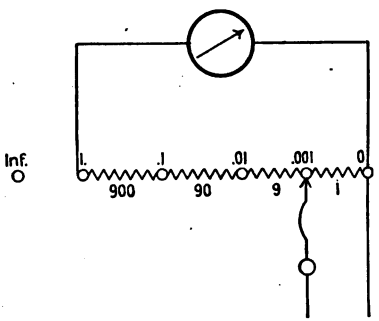



FIG. 88A.

In actual use the whole current $I$ in the circuit changes when the switch is moved from one lug to another and therefore the currents through the galvanometer do not bear to each other the ratios indicated on the lugs. But when the shunt is employed in connection with a ballistic galvanometer and a certain definite quantity of electricity is discharged from a condenser the quantities passing through the galvanometer do bear to each other the indicated ratios. Whatever the value of the current may be at any instant a definite fraction of it passes through the galvanometer and therefore the same fraction of the whole charge of the condenser must pass through it.

## EXPERIMENT 91

### CAPACITY OF A PARAFFIN PAPER CONDENSER—COMPARISON METHOD

**Apparatus.**—As in Fig. 86; paraffin paper condenser.

**Method and Manipulation.**—Charge a standard mica condenser with a battery other than a standard cell, and then discharge through a ballistic galvanometer. In the same manner charge and discharge the paraffin condenser.

Then

$$Q = CE,$$

and

$$Q_1 = C_1E;$$

$$\frac{C_1}{C} = \frac{Q_1}{Q} = \frac{d_1}{d}.$$

$$\therefore C_1 = \frac{d_1}{d} C.$$

This method should be employed whenever only a few measurements are to be taken. The figure of merit saves time in computation whenever a large number of measurements are made. In the measurement of capacities, the figure of merit is often used as explained in Exp. 88 to represent the capacity in place of the quantity of electricity for each centimeter (sometimes millimeter) of throw.

$$C = \frac{10^6 F}{E} d = fd$$

where $f$ represents the capacity in microfarads for each centimeter of throw.

Determine both the "free charge" and the "ordinary" capacity of a paper condenser.

If the condenser gives too large a throw for the galvanometer use the Ayrton shunt explained in Exp. 90.

If used at all, it must be employed with both condensers whose capacities are being compared.

In comparing capacities the magnitude of the charging electromotive force does not need to be known and therefore the standard cell should not be used. The rule is to never use a standard cell unless the known value of its e.m.f. is required.

## EXPERIMENT 92

### RESIDUAL CHARGES IN A CONDENSER

**Apparatus.**—Same as for Exp. 87; a switch.

**Method and Manipulation.**—After a condenser is discharged, it gradually acquires an appreciable charge due to the liberation of "absorbed" electricity. This effect is usually larger in paper than in mica condensers.

Connect a paper condenser to a special discharge key in the same manner as in Fig. 86, adding into the battery branch a switch for opening the battery circuit.

Charge the condenser for five minutes, and then permanently open the battery circuit by means of the switch. Discharge the condenser at a known time, and keep the discharge key down. Bring the galvanometer coil to rest, and at the end of two minutes release the key. The electricity liberated during the two minutes will then discharge through the galvanometer. At the end of another two minutes press down the key and obtain the second residual discharge. Continue in this manner until several discharges have been obtained.

Plot the throws obtained for the residual discharges on coördinate paper.

If the residual charges are small their magnitude may be increased by increasing the charging electromotive force. The original free charge will then be too large for the galvanometer unless it is shunted.

If an Ayrton shunt is used for this purpose the free charge and the residual charges can be compared.

## EXPERMENT 93

### ELECTROMOTIVE FORCE OF A CELL—CONDENSER METHOD

**Apparatus.**—As in Fig. 86; a cell whose E.M.F. is to be measured.

**Method and Manipulation**—Charge a mica condenser with the cell whose electromotive force is to be determined, and discharge through a ballistic galvanometer. Then charge the condenser with a standard cell, and discharge.

$$Q = CE,$$

and

$$Q_1 = CE_1,$$

$$\therefore \quad \frac{E}{E_1} = \frac{Q}{Q_1} = \frac{\theta}{\theta_1} \propto \frac{d}{d_1},$$

and

$$E = \frac{d}{d_1} E_1.$$

Any difference of potential may be determined in this manner. It is, therefore, possible to measure a current by this method in terms of resistance and electromotive force, or, a resistance by the fall of potential method.

## EXPERIMENT 94

### INSULATION RESISTANCE—METHOD OF LEAKAGE

**Apparatus.**—Same as in Exp. 87; a high resistance, two switches.

**Method.**—Discharge a condenser through a ballistic galvanometer and determine $Q_1$, the quantity of electricity discharged. Recharge the condenser, and allow it

to discharge through the high resistance for a known time. Then discharge through the galvanometer and determine $Q_2$, the quantity remaining in the condenser.

$-\frac{dQ}{dt} = I$, the current which flows through the high resistance $R$.

But

$$I = \frac{E}{R} = \frac{Q}{CR}.$$

$$\therefore \frac{Q}{CR} = -\frac{dQ}{dt}.$$

$$\frac{1}{R}\int_0^t dt = -C\int_{Q_1}^{Q_2} \frac{dQ}{Q},$$

$$R = \frac{t}{C \log_e \frac{Q_1}{Q_2}} = \frac{t}{2.3026C \log \frac{Q_1}{Q_2}} = \frac{t}{2.3026C \log \frac{d_1}{d_2}}.$$

If the condenser loses an appreciable part of its charge through leakage in itself, the resistance $R$, as determined above is the resistance of the high resistance and condenser in parallel. To obtain $R_x$, the value of the high resistance, determine $R_c$, the resistance of the condenser alone in the same manner as $R$ was determined. Then

$$R = \frac{R_x R_c}{R_x + R_c},$$

and

$$R_x = \frac{RR_c}{R_c - R}.$$

**Manipulation.**—The apparatus may be connected as shown in Fig. 86, except that a switch is introduced next to the battery and the high resistance to be measured is connected through another switch with the poles of the condenser. The resistance of the condenser alone is first determined.

The condenser should be charged at least five minutes on account of the absorbed charge, for this method is subject to a considerable error due to the absorption of the charge after the charging battery is disconnected. This, however, is not necessary for obtaining the value of $Q_1$ as this quantity is obtained immediately after charging. The best length of time to allow the leakage to take place is such as will make $Q_2$ one-half that of $Q_1$. This is not always practical. Ten minutes is a sufficient length of time. The high resistance is then connected in parallel with the condenser and the resistance of the two in parallel is determined in a similar manner.

1. Determine the resistance of a mica condenser.
2. Determine the resistance of some high resistance.
3. Calculate the time it takes the condenser to discharge completely through the galvanometer.
4. Calculate the time it requires the condenser to discharge all but 0.1 per cent. of its charge through the galvanometer.

## EXPERIMENT 95

### INSULATION RESISTANCE—DIRECT DEFLECTION METHOD

This experiment is the same as Exp. 75, except that a more sensitive galvanometer is employed and a higher resistance is measured. It is intended principally as a lesson to teach the use of an undamped ballistic galvanometer for current measurements.

**Apparatus.**—Ballastic galvanometer; a resistance box used as a shunt; a battery; a megohm; a high resistance of several million ohms.

**Manipulation.**—Shunt the galvanometer through a resistance that makes the motion of its coil just aperiodic.

This shunt takes the place of the damping rectangle ordinarily employed. The observations are taken in the same manner as in Exp. 75.

## EXPERIMENT 96

### WORK DONE IN CUTTING LINES OF FORCE—MAGNITUDE OF INDUCED ELECTROMOTIVE FORCE

From the definition of an electro-magnetic unit of current it follows that a wire 1 cm. in length placed perpendicular to the lines of force in a unit magnetic field, will be acted on with a force of one dyne when a unit current is flowing through the wire.

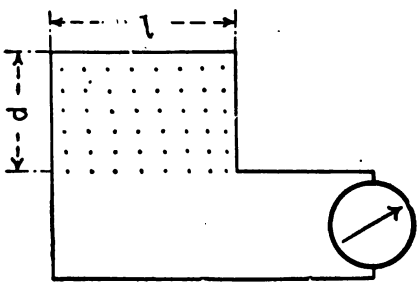



FIG. 89.

In Fig. 89 as soon as the wire $l$ is moved, it cuts lines of force, and an e.m.f. is induced in the circuit, which causes a current to flow. This current causes the field to act upon the wire, from the above consideration, with a force of $HlI'$ dynes, in a direction opposed to the motion of the wire. The mechanical work done in moving this wire through the distance $d$, is

$$W = HldI' = NI'.$$

All this mechanical work is expended in supplying the energy for the electric current, which is represented by

$$W = Q'E' = I'tE'.$$

$$\therefore I'tE' = NI',$$

and

$$E' = \frac{N}{t} = \frac{dn}{dt}. \quad E = \frac{E'}{10^8} = \frac{1}{10^8}\frac{dn}{dt}.$$

Calculate the electromotive force induced when a coil of 100 loops is cutting magnetic lines of force at the rate of 100,000 per second. How much work is done in a second in cutting the lines of force if the resistance in the circuit is 10 ohms? Get the work in ergs, joules, and foot-pounds. Show that the work in joules done in cutting magnetic lines of force is equal to the product of the electromotive force in volts, current in amperes, and the time in seconds.

Care must be taken to interpret properly the term "cutting lines of force." Would any current be induced in a loop of wire moving in a uniform field?

The letter $\Phi$ is often used in place of $N$ or $n$. The letter $N$ will always be used in this text to represent the number of lines or the flux. The letter $n$ will also be used to represent the number of loops in a coil.

## EXPERIMENT 97

### THE NUMBER OF MAGNETIC LINES OF FORCE WITHIN A SOLENOID—THE CONSTANT OF A SOLENOID

The strength of the field within a solenoid is numerically equal to the work required to move a unit magnetic pole in it through a distance of 1 cm. A unit pole has radiating from it $4\pi$ lines of force. If the solenoid is of sufficient length that all these lines can be assumed to

penetrate its loops of wire, the work done in moving the pole a distance equal to the thickness of the wire is

$$W = NI' = 4\pi I' \text{ ergs}, \qquad \text{(I)}$$

or each line of force will have cut the wire once. In moving the distance of one centimeter, each line cuts $\frac{n}{l}$ wires, so that the work done is

$$W = H = NI'\frac{n}{l} = \frac{4\pi n}{l}I'. \qquad \text{(II)}$$

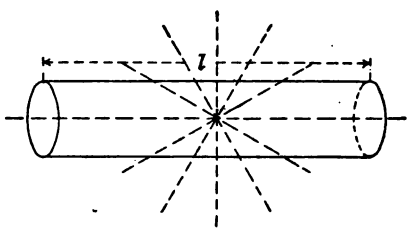



FIG. 90.

If the number of lines passing through the ends of the solenoid is not negligible, but still is small, a correction can be made in the following manner:

The surface area of a sphere circumscribed about a solenoid is $4\pi R^2$, and the area of the two segments cut off by the ends of the solenoid is approximately $2\pi r^2$. The number of lines from the unit pole that penetrate the loops of wire is

$$N' = 4\pi\left(1 - \frac{2\pi r^2}{4\pi R^2}\right) = 4\pi\left(1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right).$$

The strength of the field near the center of the solenoid therefore is

$$H = \frac{4\pi n}{l}\left(1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right)I'; \qquad \text{(III)}$$

and the total number of lines within the central portion of the solenoid is

$$N = HA = \frac{4\pi nA}{l}\left(1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right)I' = K'I' = \frac{K'}{10}I = KI, \quad \text{(IV)}$$

$$K = \frac{2\pi nA}{5l}\left(1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right). \quad \text{(V)}$$

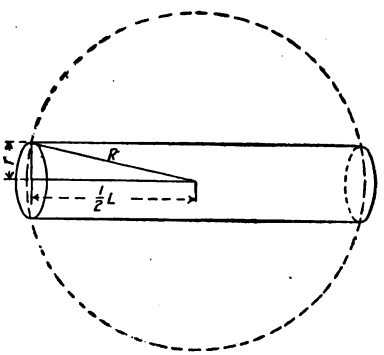



FIG. 91.

The constant $K$ represents the number of lines threading through the center portion of the solenoid when the current flowing is one ampere, and $A$ is the area of the cross-section, the diameter being measured from center to center of wire.

An equation[1] for the intensity of the magnetic field at any point of the axis of a solenoid is

$$H = \frac{2\pi nI}{10l}(\cos\alpha + \cos\alpha'),$$

where the angles are those indicated in Fig. 92.

[1] Stewart and Gee, Vol. 2, p. 329.

The intensity of the field at the center of the solenoid from this equation is

$$H = \frac{2\pi nI}{5l} \cos \alpha,$$

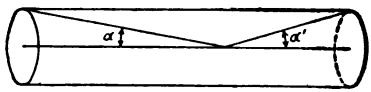



FIG. 92.

and

$$N = HA = \frac{2\pi nAI}{5l} \cos \alpha = KI,$$

$$K = \frac{2\pi nA}{5l} \cos \alpha. \qquad \text{(VI)}$$

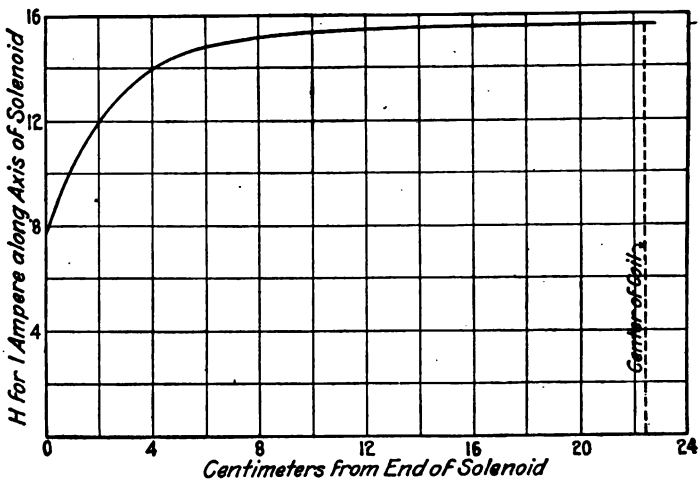



FIG. 93.

Both of these formulæ are only close approximations. When the coil has more than one layer of wire, the values of $K$ or $N$ are the sum of the values for the separate layers.

The intensity of the magnetic field due to one ampere within U. of M. solenoid No. 5, at different points on

the axis, extending from one end to the center, is shown in Fig. 93.

Calculate $K$ for the solenoid assigned to your table.

## EXPERIMENT 98

### CALCULATION OF THE COEFFICIENT OF MUTUAL INDUCTION OF A CURRENT INDUCTOR

The coefficient of mutual induction is the ratio between the induced electromotive force in the secondary and the rate of change of the current in the primary.

$$M = \frac{E}{\frac{di}{dt}}.$$

Imagine the current in the primary to be changing at the rate of one ampere per second. Then the coefficient of mutual induction is numerically equal to the

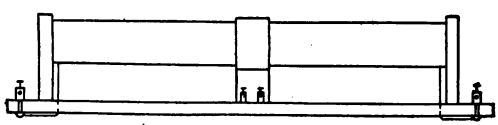


FIG. 94.

induced electromotive force. The magnetic lines of force due to one ampere then close up in one second, if the current is diminishing; or pass outward, if the current is increasing; in either case all these lines, if the secondary is wound over the central portion of a long solenoid, cut the loops of the secondary coil. The number of these lines, $K$, produced by each ampere, can be calculated by means of one of the equations in Exp. 97. The induced electromotive force $E_1'$, in each loop of the secondary is numerically equal to the number of lines that cut

it per second, which in this case is $K$. The electromotive force in volts

$$E_1 = \frac{E_1'}{10^8} = \frac{K}{10^8},$$

and

$$M = n_2 E_1 = \frac{n_2 K}{10^8},$$

where $n_2$ is the number of turns in the secondary coil.

Calculate the coefficient of mutual induction of a current inductor.

## EXPERIMENT 99

### RELATION BETWEEN THE QUANTITY OF INDUCED ELECTRICITY IN THE SECONDARY CIRCUIT AND THE CURRENT IN THE PRIMARY

Arrange the apparatus as shown in Fig. 95. The galvanometer circuit containing the secondary of the

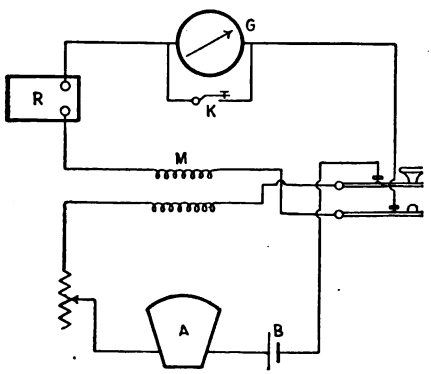



FIG. 95.

mutual induction coil or current inductor $M$ is opened immediately after the opening of the primary circuit. Read the ammeter, and observe the throw on the ballistic

galvanometer when the circuits are opened. Repeat for several different currents. Determine the relation between $Q$ and $I$ by plotting on coördinate paper. The throws may be plotted in place of the quantities.

Whenever it is possible, the resistance in the galvanometer circuit should not be less than that which makes the coil just aperiodic.

The galvanometer circuit, when closed, may contain a thermo-electric current so that the coil is not at rest at the true null point.

The magnitude of the throw must be reckoned from the null point as it is on open circuit. This is similar to the case in Exp. 86 where the throws are taken from the elongation of a vibrating coil.

## EXPERIMENT 100

### RELATION BETWEEN THE QUANTITY OF INDUCED ELECTRICITY AND THE RESISTANCE IN THE SECONDARY CIRCUIT

Use the same apparatus as in Exp. 99. The current in the primary circuit is kept constant while the resistance in the secondary is varied The throw on the ballistic galvanometer is observed for each of several resistances when the circuits are opened. The total resistance $R_s$ in the secondary circuit must be known for each of the observations. Determine the relation between $Q_s$ and $\frac{1}{R_s}$ by plotting, showing that the induced quantity varies inversely as the resistance.

The observed throws may be plotted in place of the quantities of electricity.

The resistance $R_s$ includes the resistance of the box, the galvanometer, and the secondary coil. Its largest value should be at least four times that of the smallest.

## EXPERIMENT 101

### TO MEASURE COEFFICIENT OF MUTUAL INDUCTION

From the definition of the coefficient of mutual induction

$$M = \frac{E_s}{\frac{di}{dt}} = \frac{E_s}{\frac{I_p}{t}} = \frac{I_s t R_s}{I_p} = \frac{Q_s R_s}{I_p},$$

where $R_s$ is the total resistance in the secondary circuit, and $Q_s$ the quantity of induced electricity in the secondary when the primary circuit containing the current $I_p$ is either closed or opened. From this equation

$$Q_s = M \frac{I_p}{R_s},$$

which shows the relations proven experimentally in Exps. 99 and 100.

Measure the coefficient of mutual induction of the current inductor of Exp. 98, and compare with the calculated results.

Determine the value of $M$, also, from the observations of Exp. 100.

## EXPERIMENT 102

### CALCULATION OF THE COEFFICIENT OF SELF-INDUCTION

The coefficient of self-induction is the ratio of the electromotive force induced in a coil to the rate at which the current in it is changing.

$$L = \frac{E}{\frac{di}{dt}}.$$

This is similar to the definition of the coefficient of mutual induction, except that in this case the magnetic

lines of force cut the loops of the coil that produces them. The coefficient of self-induction is numerically equal to the induced electromotive force when the current is changing at the rate of one ampere per second.

If all the lines that pass through the central section of a solenoid passed out of the ends, then when the current is changing at the rate of one ampere per second $K$ lines of force would cut each loop of the coil. The induced electromotive force then would be

$$E = \frac{E_1'}{10^8} = \frac{n_1 K}{10^8},$$

where $n_1$ is the number of loops and $K$ the constant of the solenoid.

Therefore,

$$L = E_1 = \frac{n_1 K}{10^8}. \qquad \text{(I)}$$

Since about one-half of the lines pass out of the solenoid before reaching the ends, as was shown in Fig. 93, Exp. 97, this equation gives only a rough approximation. The following is a more exact formula:

$$L = \frac{4\pi^2 n^2}{10^9}\left[\frac{2a^4 + a^2 l^2}{\sqrt{4a^2 + l^2}} - \frac{8a^3}{3\pi}\right]^1, \qquad \text{(II)}$$

in which $a$ = radius, $l$ = length of coil, and $n$ = number of turns of wire per centimeter.

## EXPERIMENT 103

### COEFFICIENT OF SELF-INDUCTION—ZELENY'S MODIFICATION OF MAXWELL'S METHOD[2]

The coil $R_1$, whose coefficient of self-induction is to be measured, is connected to three non-inductive

[1] Bulletin Bureau of Standards, Vol. III, p. 303, 1907; Vol. IV, p. 385, 1907–8.

[2] Physical Review, Vol. XXIV, p. 260, 1907.

resistances with which it forms a Wheatstone bridge as shown in Fig. 96. When ordinary resistance boxes are used for the non-inductive resistances, a sliding resistance and a resistance box containing tenths of ohms are placed in the same arm of the bridge with the coil whose coefficient of self-induction is to be measured. This makes possible an exact balance of the resistances in the four arms.

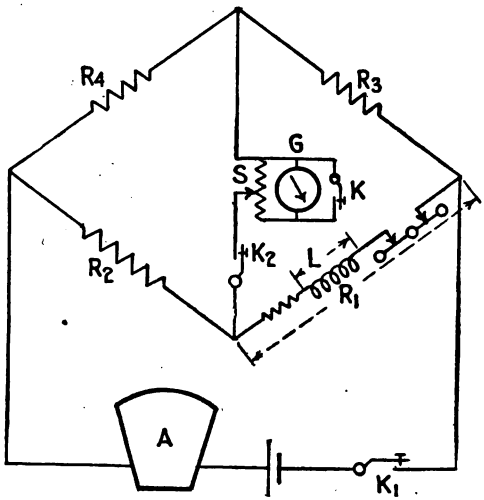



FIG. 96.

The current $I$ in the main circuit is measured after the balance of the resistances is obtained, and from this the current $I_1$ flowing through $R_1$ is calculated.

$$I_1 = \frac{(R_3 + R_4)I}{R_1 + R_2 + R_3 + R_4}. \qquad (I)$$

The circuit is then opened, and the induced electromotive force in $R_1$ causes an instantaneous current to flow through the resistances and the ballistic galvanometer. The whole quantity $Q$ of the induced electricity

passes through $R_1$ and $R_3$, but it divides into two parts in passing through the galvanometer and the resistance $R_2 + R_4$. The part passing through the galvanometer

$$q = \frac{(R_2 + R_4)Q}{R_2 + R_4 + G},$$

so that

$$Q = \frac{R_2 + R_4 + G}{R_2 + R_4} q. \qquad \text{(II)}$$

The quantity $q$ is obtained from the throw of the galvanometer coil and the figure of merit on open circuit, the keys $K_1$ and $K_2$ being opened in rapid succession.

The induced electromotive force is situated in $R_1$, so that the resistance of the Wheatstone net for the induced electricity is

$$R = R_1 + R_3 + \frac{(R_2 + R_4)G}{R_2 + R_4 + G}. \qquad \text{(III)}$$

$$L = \frac{e}{\frac{di}{dt}} = \frac{iR}{\frac{I_1}{t}} = \frac{itR}{I_1} = \frac{QR}{I_1}. \qquad \text{(IV)}$$

The values of $Q$, $R$ and $I_1$ can be calculated from Equations (I), (II) and (III).

It is often convenient to make $R_2 = R_4 = 100$ ohms, and to introduce a resistance box with tenths of ohms into the inductance branch, so that $R_3$ may be made some multiple of ten. In the equations, $R_1$ is the sum of all resistances in that branch. The resistance $G$ represents the sum of all the resistances in the galvanometer branch.

While obtaining a balance, employ a shunt with the galvanometer. For the final observations the shunt circuit usually should be opened. If this is not the case, care must be taken to obtain the figure of merit of the galvanometer with the same resistance in the shunt.

The self-induction in the galvanometer coil does not affect this equation, for when the current in the main circuit is broken the induced current caused to flow through the galvanometer increases from zero to a maximum and then decreases to zero. The quantity of electricity induced in the galvanometer coil while the transient current in it is increasing, is equal to the quantity induced while the current is decreasing. The

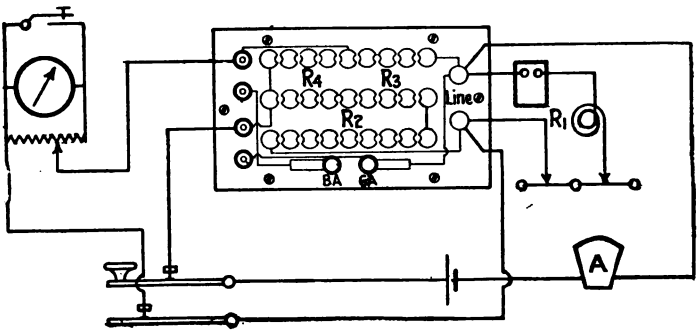



FIG. 97.

two quantities are, however, sent in opposite directions and their algebraic sum being zero they do not affect the throw of the galvanometer coil.

It is often most convenient to use the three resistances of a Wheatstone box bridge. The connections are made as shown in Fig. 97, the keys on the box bridge not being used.

The heating of the copper coil often alters the balance of the bridge quite rapidly making it necessary to move the adjusting slide continually. The throws are measured from the zero on open circuit.

## EXPERIMENT 104

### COEFFICIENT OF SELF-INDUCTION—ANDERSON'S METHOD[1]

This method of measuring self-inductance has an advantage over the method of the previous experiment in that it is a null method and does not require the measurement of the current and of the quantity of electricity. The coil whose inductance $L$ is to be measured is placed

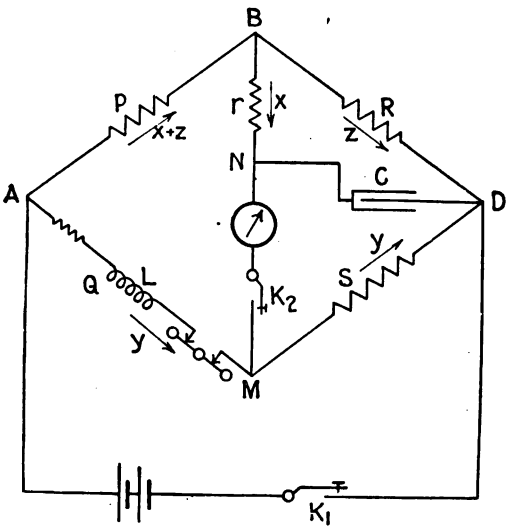



FIG. 98.

in one arm of the Wheatstone bridge together with a box containing tenths of ohms and with a sliding resistance. The resistance $r$ is placed in the galvanometer branch, and the condenser $C$ is connected as shown in Fig. 98.

[1] Philosophical Magazine, Vol. XXXI, p. 329, 1891. Physical Review, Vol. XXIV, p. 265, 1907.

As in the previous method, it is often convenient to employ the resistances of a box bridge. The connections can be made as shown in Fig. 99.

The resistance $R$ may be made equal to $P$. The bridge is balanced with a continuous current as in Exp. 103. The resistance $r$ or the capacity $C$, or both, are then adjusted until no throw is produced on the galvanometer when the key for opening the circuits is depressed. The induced electromotive force in $Q$ and the

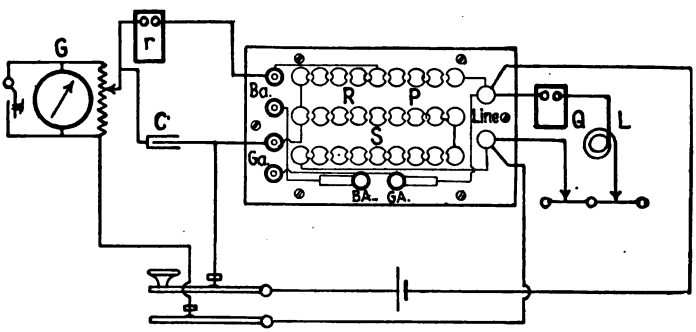



FIG. 99.

discharge from the condenser act in opposite directions through the galvanometer, and with the proper value of $r$ the two effects are equal so that there is no difference of potential between the extremities $M$ and $N$ of the galvanometer. The condition is the same at both the opening and the closing of the circuits. The bridge is then balanced for both continuous and intermittent currents.

The condition for a balance with continuous current is that

$$\frac{P}{R} = \frac{Q}{S} \qquad (1)$$

The additional condition for a balance with intermittent currents is that there shall be no difference of potential between $M$ and $N$ at the opening or closing of the circuits. Let $x$ be the quantity of electricity that has flowed into the condenser during the time $t$ after the closing of the circuits, and $z$ the quantity that has passed through $R$. Then $x + z$ has passed through $P$. Also let $y$ be the quantity that has passed through $Q$ and $S$ during the same time. Since $M$ and $N$ have the same potential, the difference of potential between the ends of the condenser must equal the difference of potential between the extremities of $S$, so that

$$\frac{x}{C} = S\frac{dy}{dt}, \qquad \text{(II)}$$

where $\frac{dy}{dt}$ is the current flowing through $S$.

The fall of potential from $B$ to $D$ must be the same by both paths, so that

$$R\frac{dz}{dt} = \frac{x}{C} + r\frac{dx}{dt}. \qquad \text{(III)}$$

The fall of potential from $A$ through $B$ to $N$ must be equal to that from $A$ to $M$. The latter contains the inductance coil, the induced electromotive force in which is equal to $L\frac{d(dy)}{dt} = L\frac{d^2y}{dt^2}$.

Then

$$r\frac{dx}{dt} + P\left(\frac{dx}{dt} + \frac{dz}{dt}\right) = Q\frac{dy}{dt} + L\frac{d^2y}{dt^2}. \qquad \text{(IV)}$$

Substituting from equations (II) and (III) in (IV),

$$(r + P)\frac{dx}{dt} + \frac{P}{R}\left(\frac{x}{C} + r\frac{dx}{dt}\right) = \frac{Q}{S}\cdot\frac{x}{C} + \frac{L}{SC}\cdot\frac{dx}{dt}. \qquad \text{(V)}$$

$$(r + P)\frac{dx}{dt} + \frac{Pr}{R}\frac{dx}{dt} + \frac{x}{C}\left(\frac{P}{R} - \frac{Q}{S}\right) = \frac{L}{SC}\cdot\frac{dx}{dt}. \qquad \text{(VI)}$$

This equation represents the conditions necessary for a balance with intermittent or variable currents. The Wheatstone net is at the same time balanced also for continuous currents so that from equation (I),

$$\frac{P}{R} - \frac{Q}{S} = 0.$$

Then equation (VI) becomes

$$r + P + \frac{Pr}{R} = \frac{L}{SC},$$

from which since $PS = QR$,

$$L = C\{r(Q + S) + PS\}. \qquad \text{(VII)}$$

For sensitiveness of the final adjustment it is desirable to make $R$ and $S$ large, and $r$ small. Since $Q$ is usually small, $P$ will also be small. When the maximum sensitiveness is not required, it is often advantageous to make $R = P = 10$ or 100. This makes $r$ larger which is a condition convenient for adjustment.

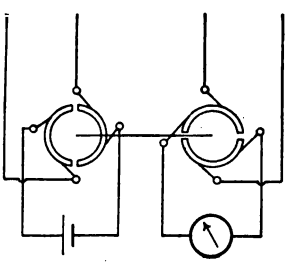


FIG. 100.

If thermo-electric currents are present in the galvanometer circuit they must either be eliminated by a potentiometer method or proper allowance must be made for them when balancing the bridge.

For the measurement of very small self-inductances, a double rotating commutator is usually employed. It consists of two two-part commutators mounted on a common shaft, usually on opposite ends of the shaft of a small motor. The battery is connected through one commutator while the galvanometer is connected through the other, as shown in Fig. 100. The commuta-

tors are so placed that when one causes an alternating current to flow through the arms of a bridge, the other so commutates the alternating current in the galvanometer branch as to send a succession of unidirectional pulses through the galvanometer. This can be made to increase the sensitiveness of the method about one hundred times. A rapid succession of pulses acts upon the galvanometer in place of one.

Another form of a rotating commutator may be employed which sends pulses through the galvanometer only at the break of the circuit, the galvanometer being on open circuit a greater part of the time between successive pulses.

This method is also used for measuring capacities in terms of a known self-inductance.

From equation (VII)

$$C = \frac{L}{r(Q + S) + PS}. \qquad \text{(VIII)}$$

## EXPERIMENT 105

### COMPARISON OF TWO COEFFICIENTS OF SELF-INDUCTION

The apparatus is connected as shown diagrammatically in Fig. 101. The battery and galvanometer are usually connected through a double key and preferably through a double rotating commutator. A sliding resistance and a box with tenths of ohms, not shown in the figure, are usually placed in any one of the arms for convenience of adjustment.

Let $R_1$ and $R_2$ have the coefficients of self-induction $L_1$ and $L_2$, and let $R_3$ and $R_4$ be inductionless resistances. One of the inductances $L_2$ is a variable standard.

The resistances $R_3$ and $R_4$ are adjusted to give a balance with steady current. The variable standard is

then adjusted until a balance is obtained for intermittent currents also. This adjustment does not alter the resistances in the bridge.

Let $i_1$ be the current through $R_1$ and $R_2$, and $i_3$ that through $R_3$ and $R_4$, at the same instant $t$ after closing

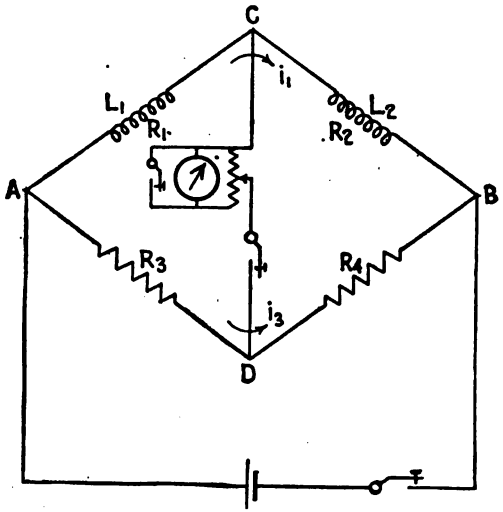



FIG. 101.

the circuit. Since no current flows through the galvanometer the potential at $C$ must be the same as that at $D$. Hence

$$R_1 i_1 + L_1 \frac{di_1}{dt} = R_3 i_3, \qquad \text{(I)}$$

$$R_2 i_1 + L_2 \frac{di_1}{dt} = R_4 i_3. \qquad \text{(II)}$$

From the condition of steady current,

$$R_1 R_4 = R_2 R_3. \qquad \text{(III)}$$

From equations (I) and (II),

$$R_1 R_4 i_1 i_3 + L_1 R_4 \frac{di_1}{dt} i_3 = R_2 R_3 i_1 i_3 + L_2 R_3 \frac{di_1}{dt} i_3. \qquad \text{(IV)}$$

From the relationship of equation (III), equation (IV) becomes after cancellation,

$$L_1 R_4 = L_2 R_3,$$

and

$$L_1 = \frac{R_3}{R_4} L_2. \qquad \text{(V)}$$

The variable standard of self-induction contains two coils without iron joined in series, one of them fixed and the other movable about a vertical axis. The self-induction of the two depends upon their relative position. A graduated scale indicates the inductance in millihenries for any position of the movable coil.

## EXPERIMENT 106

### ABSOLUTE DETERMINATION OF RESISTANCE—STANDARD SOLENOID OR CURRENT INDUCTOR METHOD

A current inductor consists of a standard solenoid with a secondary coil of $n_2$ turns as shown in Fig. 94, Exp. 98.

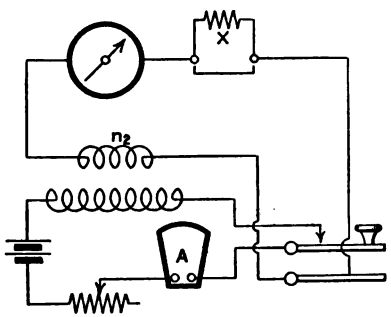



FIG. 102.

These coils are connected as shown in Fig. 102. The "unknown" resistance, $X$, for the purpose of checking the result easily may be a 200-ohm coil in a resistance box. The current by means of a single tap on the double

key is broken an instant before the secondary circuit is opened.

The resistance $X$, to be measured, can be placed into or taken out of the galvanometer circuit. Throws are obtained with this resistance in and out of the circuit.

$$E = \frac{1}{10^8}\frac{dn}{dt} = \frac{Nn_2}{10^8 t} = IR.$$

From which

$$R = \frac{Nn_2}{10^8 Q_1} = \frac{KI_1 n_2}{10^8 Q_1}. \quad \text{[See equation (IV), Exp. 97.]}$$

In a similar manner when $X$ is in the galvanometer circuit,

$$R + X = \frac{Nn_2}{10^8 Q_2} = \frac{KI_2 n_2}{10^8 Q_2}.$$

Then

$$X = \frac{KI_2 n_2}{10^8 Q_2} - \frac{KI_1 n_2}{10^8 Q_1} = \frac{Kn_2}{10^8}\left(\frac{I_2}{Q_2} - \frac{I_1}{Q_1}\right) = \frac{Kn_2}{10^8 F}\left(\frac{I_2}{d_2} - \frac{I_1}{d_1}\right),$$

where $K$ is the constant of the solenoid, $n_2$ the number of turns in the secondary coil, and $F$ the figure of merit of the galvanometer.

The resistance of the circuit should not be less than that which makes the coil just aperiodic. The currents $I_1$ and $I_2$ should have such magnitudes as to make the throws $d_1$ and $d_2$ nearly equal.

The throws must be reckoned from the zero on open circuit.

## EXPERIMENT 107

### FIGURE OF MERIT—BY FOUR METHODS

The figure of merit of a galvanometer is usually determined by one of the following four methods:

**1. Condenser Method.**

$$F = \frac{Q}{d} = \frac{CE}{d}. \qquad \text{(I)}$$

This is given in Exp. 88 and is the method commonly employed for determining the figure of merit on open circuit. Only the free charges of the condenser can be used as explained in Exp. 87.

This method is not commonly used for determining the figure of merit on closed circuit. It may, however, be employed if proper precautions are taken. The condenser is discharged through the galvanometer which,

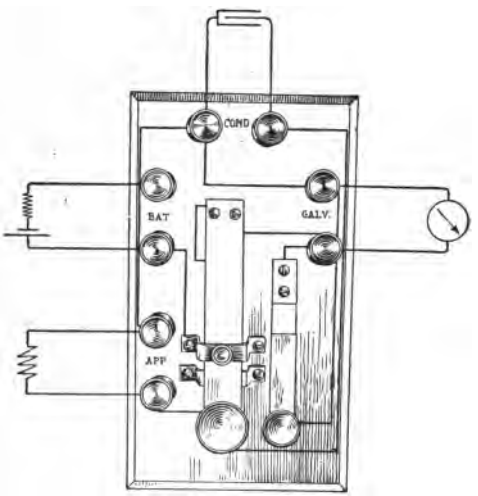



FIG. 103.

before the coil moves an appreciable distance, is placed on closed circuit with the apparatus with which it is to be used. This can be approximately accomplished by means of one tap on the key shown in Fig. 103. The key is held down until the reading of the throw is obtained. An error is often produced by the coil moving an appreciable distance before the circuit is closed. To reduce this error the circuit should have a resistance not less than that which makes the galvanometer just

aperiodic, and the tap on the key should be rapid. The greater the period of the galvanometer, the less is the danger of error from this cause.

A thermo-electric current in the circuit does not produce its full deflection during the period of the throw. Only a fraction[1] of the full thermo-electric deflection

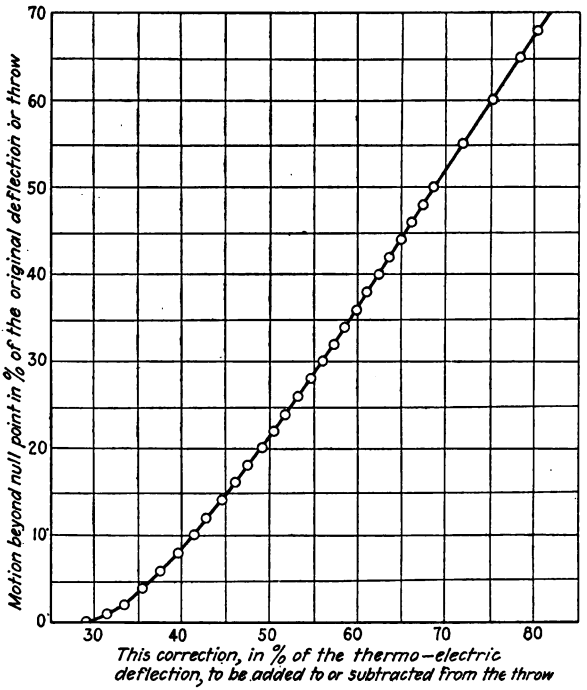



FIG. 104.

is effective, and this fraction depends on the damping of the galvanometer and can be obtained from the curve of Fig. 104. After an observed throw, the coil is allowed to swing beyond the zero point, on the closed circuit.

[1] Physical Review, Vol. XXIII, p. 414, 1906.

The distance it moves beyond the zero gives the necessary datum for obtaining the correction from the curve and the observed thermo-electric deflection.

**2. Standard Mutual Inductance Method.**—This method is particularly adapted for determining the figure of merit on closed circuit. The secondary coil is made a part of the galvanometer circuit. From Exp. 101 it follows that

$$F = \frac{Q}{d_s} = \frac{MI_p}{Rd_s}. \qquad \text{(II)}$$

The figure of merit on open circuit may be determined by this method by opening the secondary circuit immediately after the opening of the primary circuit. This is best accomplished by means of a key as shown in Fig. 95, Exp. 99. The resistance of the galvanometer circuit should not be less than that which makes the coil just aperiodic. The standard mutual induction coil is a set of two coils whose coefficient of mutual induction is known, being given by the manufacturer or having been previously determined.

**3. Current Inductor or Standard Solenoid Method.**—This method is the same as the standard mutual induction coil method except that the primary coil is a standard solenoid whose constant can be calculated.

$$F = \frac{Q}{d_s} = \frac{MI_p}{Rd_s}\,\frac{n_s K I_p}{10^8 Rd_s}. \qquad \text{[See Exp. 98.]} \quad \text{(III)}$$

**4. Hibbert Magnetic Standard Method.**—The Hibbert magnetic standard consists of a radial magnetic field of known number of lines and a coil of known number of turns. The coil is dropped through the field cutting a known number of lines of force. It is employed for closed circuit work only and is the most convenient of all the standards for such work in many cases.

From Exp. 96,

$$E = \frac{1}{10^8}\frac{dn}{dt} = \frac{Nn}{10^8 t} = IR.$$

From which,

$$N_s n_s = 10^8 ItR = 10^8 QR, \qquad \text{(IV)}$$

$$Q = \frac{N_s n_s}{10^8 R},$$

$$F = \frac{Q}{d_s} = \frac{N_s n_s}{10^8 R d_s}. \qquad \text{(V)}$$

Make the following determinations:

1. The figure of merit by each of the four methods for a circuit in which the galvanometer coil is just aperiodic. Compare the four values. Record the resistance of the circuit.

2. Determine the figure of merit on open circuit by the condenser method, for throws of different magnitude. Plot the values.

3. Determine the figure of merit on closed circuit by the standard mutual inductance method, for different known resistances of the circuit. Use such a current in the primary as will produce a throw of approximately 10 cm. for each observation. Plot the values.

4. Determine the figure of merit on closed circuit by means of the Hibbert standard, for different known resistances of the circuit. Plot the values on the same paper with those of the last set and compare.

## EXPERIMENT 108

### COMPARISON METHOD FOR FLUX MEASUREMENTS—EXPLORING COILS

In making measurements of magnetic flux and the intensity of magnetic fields an exploring coil is connected in series with a ballistic galvanometer, a resistance box,

and the secondary coil of a standard inductance apparatus, as shown in Fig. 105. The exploring coil has $n$ loops all of which cut the lines of force when it is withdrawn from the field. The throw $d$ produced on the galvanometer is compared with the throw obtained from the standard inductance. The resistance of the galvanometer circuit is the same in both cases and cancels in the equation. In the following discussion $n_s$ and $d_s$ refer to the standard.

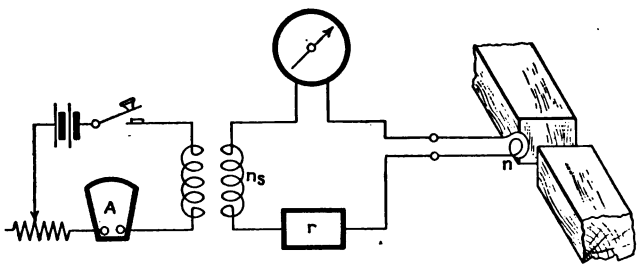



FIG. 105.

If $N$ is the number of lines of force cut by the number of loops $n$ of the exploring coil, equation (IV), Exp. 107 gives

$$Nn = 10^8QR, \qquad \text{(I)}$$

in which $Q$ is the quantity of electricity induced in coulombs and $R$ the total resistance in ohms in the galvanometer circuit.

From equation (I)

$$N = \frac{10^8QR}{n} = \frac{10^8FdR}{n}. \qquad \text{(II)}$$

From this equation it is seen that if the figure of merit is known for the particular circuit, the number of lines cut by each loop of the coil can be calculated. The only troublesome factor is the resistance of the circuit, $R$, which however, can be cancelled by substituting for the figure its expression derived in Exp. 107. The

substitutions for each of the three applicable standards are as follows:

1. *Standard mutual inductance method:*

$$F = \frac{MI_p}{Rd_s}.$$

$$N = \frac{10^8 MI_p}{n} \frac{d}{d_s}. \qquad \text{(III)}$$

2. *Standard solenoid method:*

$$F = \frac{n_s K I_p}{10^8 R d_s}.$$

$$N = KI_p \frac{n_s}{n} \frac{d}{d_s}. \qquad \text{(IV)}$$

3. *Hibbert standard method:*

$$F = \frac{N_s n_s}{10^8 R d_s}.$$

$$N = N_s \frac{n_s}{n} \frac{d}{d_s}. \qquad \text{(V)}$$

By means of either of these equations (III), (IV) and (V) the number of lines within the loop of the exploring coil before it is jerked from the field, can be determined. The only observations necessary are the two throws, making this for all practical purposes a comparison method.

If another type of exploring coil is used in which the loops are rotated through an angle of 180° in place of being jerked from the field, the lines of force within the coil are cut twice, and the throw obtained is twice what it would be if the coil were of the first type. When using an exploring coil that rotates through 180° the values of $N$ obtained by means of the equations (III), (IV) and (V) must therefore be divided by the factor 2.

To obtain the intensity of the field,

$$F = \frac{N}{A},$$

where $A$ is the average cross-section of the loops through which the number of lines $N$ pass.

## EXPERIMENT 109

### MEASUREMENT OF MAGNETIC FIELDS AND POLE STRENGTHS

1. Measure the intensity of the field between the poles of an electromagnet, by means of an exploring coil described in Exp. 108, for various strengths of the exciting current up to 5 amperes. First measure any residual magnetization that may be present and then increase the current by steps measuring the strength of the field at each step. Then decrease the current and take measurements in the same manner. The resistance in the circuit should be such as to make the galvanometer just aperiodic. Plot the observations placing the strength of the magnetizing current on the axis of abscissas. Observe the hysteresis.

2. Determine the number of lines about the pole of a magnet by cutting them with a coil of wire, and then calculate the pole strength. Place the coil about the center of the magnet and withdraw one of the two with a jerk, taking the galvanometer throw.

$$\text{Pole strength} = \frac{N}{4\pi}.$$

## EXPERIMENT 110

### HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FIELD—EARTH INDUCTOR METHOD

The earth inductor consists of a coil of wire that can be rotated in the earth's field. In Fig. 106 the coil is wound on the circular frame. The square frame can be placed in any position and clamped. The circular

frame rotates on an axis perpendicular to the axis of the square one.

The square frame is clamped in a vertical position and by the aid of a magnetic needle its plane is placed perpendicular to the magnetic meridian. The coil is then connected in series with a ballistic galvanometer, a standard mutual inductance coil and sufficient resistance to make the galvanometer approximately just aperiodic.

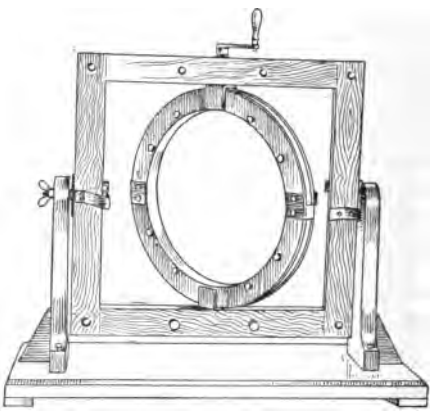


FIG. 106.

The frame of the coil is placed into the plane of the square frame, and then is rotated quickly through half a revolution. The quantity of induced electricity is obtained from the throw by comparison with the throw obtained from the standard mutual inductance coil.

The number of lines of force passing through the coil in its initial position is $AH$, where $A$ is the average area of the loops in the coil. In turning through 180° these lines are cut twice.

The earth inductor is a large exploring coil of the rotating type. The total number of lines passing through the

coil in any position can be measured by use of any one of the three equations, (III), (IV), (V), of Exp. 108.

When using the mutual inductance standard

$$N = \frac{10^8 M I_p}{2n} \frac{d}{d_s}.$$

The factor 2 appears because the throw $d$ is due to the lines having been cut twice.

$$H = \frac{N}{A} = \frac{10^8 M I_p}{2nA} \frac{d}{d_s}.$$

## EXPERIMENT 111

### INCLINATION OF THE EARTH'S MAGNETIC FIELD—EARTH INDUCTOR METHOD

The earth inductor is adjusted in a vertical plane as in Exp. 110, except that the standard inductance may be removed. The coil is turned quickly through 180°, and the throw on the galvanometer is noted. The square frame is then clamped in a horizontal position, and the throw is again taken for half a turn of the coil.

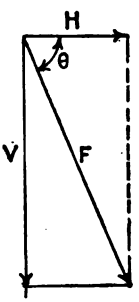


FIG. 107.

In the first case, from equation (IV), Exp. 107,

$$Q_h = \frac{Nn}{10^8 R} = \frac{2AHn}{10^8 R}.$$

In the second case, the vertical component $V$ of the earth's field is effective. Then

$$Q_v = \frac{2AVn}{10^8 R}.$$

Then

$$\frac{d_v}{d_h} \propto \frac{Q_v}{Q_h} = \frac{V}{H} = \tan \theta,$$

from which the angle of inclination, $\theta$, is calculated.

## EXPERIMENT 112

### INTENSITY OF THE EARTH'S MAGNETIC FIELD

When the inclination and $H$ are known, the intensity of the earth's field

$$F = \frac{H}{\cos \theta}.$$

Compute from the observations of Exps. 110 and 111.

## EXPERIMENT 113

### MAGNETIC PROPERTIES OF IRON—I, B, $\mu$, R AND HYSTERESIS

**Apparatus.**—In Fig. 108, $S$ is a standard mutual inductance coil for obtaining the constant of the galvanometer on closed circuit, $T$ is a solenoid placed perpendicular to the magnetic meridian, which has the number of turns of wire known for both its coils, $U$ is a set of two coils one sliding over the other for the purpose of varying the inductance.

The piece of iron to be tested should be in the form of a wire or several fine wires somewhat shorter than the primary coil of $T$.

**Manipulation.**—Adjust the coils of $U$ until the inductance is equal and opposed to the inductance in $T$, and with the largest current to be employed no throw is produced on the galvanometer on closing or opening the circuit of battery $B$.

Place the iron into the primary coil of $T$ when the battery circuit is open. By means of a copper wire or a string attached to the iron quickly withdraw the iron from the coil. This should produce no throw on the galvanometer. If it does, try to remove the magnetism

from the iron, and if any finally remains, take the initial magnetism into consideration as a part of the total magnetism that will be induced in the iron. Care must be taken to make the correction in the proper sense.

With the iron in $T$, increase the current by equal steps until finally a considerable increase produces but little effect upon the galvanometer, then, in the same manner,

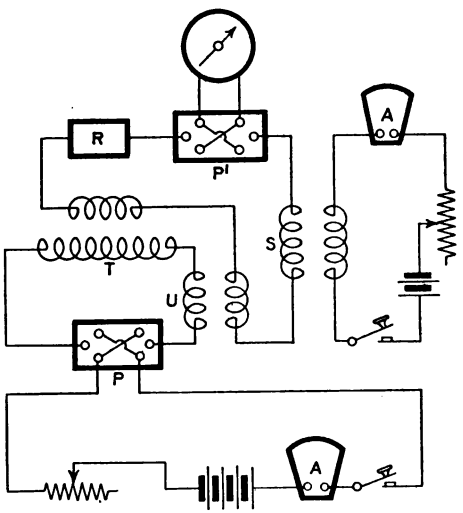



FIG. 108.

decrease the current to zero. Then reverse the commutators $P$ and $P'$ and repeat the process, and then again reverse and increase the current to the maximum. The ballistic throw and the ammeter reading are taken at each increase. These give the data for calculating the increases in the magnetizing force and the induction.

Obtain throws from the standard mutual inductance coil. Large throws should be compared with large, and

small with small. Any method of Exp. 108 may be employed.

*H.—Magnetizing force.*

$$H = \frac{4\pi nI}{10l}\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right].$$

*$N_x$.—Number of lines induced at each increase of the magnetizing force.*

$$N_x = N'' - N' = \frac{10^8 QR}{n_2}.$$

This number, $(N'' - N')$, is the number $N$ of equation (II), Exp. 108, and is determined in the same manner.

*I.—Intensity of magnetization.*

$$I = \frac{M}{V} = \frac{ml}{sl} = \frac{4\pi m}{4\pi s} = \frac{N}{4\pi s},$$

where $N$ is the total number of lines in the iron, and $s$ the cross-section of the iron in square centimeters. The number of induced lines per square centimeter,

$$\frac{N}{s} = 4\pi I.$$

*B.—Magnetic induction* refers to the number of lines of force per square centimeter through the iron. It includes the lines due to the magnetizing force and the induced lines.

$$B = H + 4\pi I.$$

*K.—Magnetic susceptibility* is the ratio of the intensity of magnetization to the magnetizing force.

$$K = \frac{I}{H}.$$

*Magnetic permeability* is the ratio of the magnetic induction to the magnetizing force.

$$\mu = \frac{B}{H}.$$

*Magnetic reluctance* $R = \frac{l}{\mu s}$.

1. Tabulate the values of $H$ and $B$ for all the observed points, and plot the values of $H$ as abscissas and those of $B$ as ordinates. The first branch is called the magnetization curve, and the closed part, the hysteresis curve.

Since $H$ is proportional to the magnetizing current and $B$ to the algebraic sum of the throws, the nature of the hysteresis curve can be obtained by plotting the currents with the sums of the throws. When the rocker of the commutator leading to the galvanometer is reversed, the sign of the throws is reversed.

2. Tabulate the values of $H$, $N$, $I$, B, $\mu$, and $R$ for a few important points on the hysteresis curve. There are two values for these quantities for each value of $H$. The direction of the quantities is designated by + and − signs.

These values are not exactly the same as would be obtained with the iron made in the form of a ring. The reluctance is that of the iron and the air through which the lines of force pass. This influences all the other quantities. When the wire is long compared to its diameter, these values are nearly the same as those obtained with a ring.

# SOUND

## EXPERIMENT 114

### VELOCITY OF A TRANSVERSE WAVE ALONG A CORD

**Apparatus.**—A cord about 20 meters long, and of about 4 mm. diameter, has one end attached to a rigid support. At the other end, the cord passes over a pulley and supports a weight that may be varied from one to four kilograms. The pulley should have ball bearings so that the friction may be as small as possible.

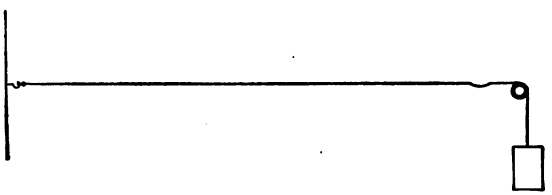


FIG. 109.

**Method and Manipulation.**—Attach a weight of approximately one kilogram to the end of the cord. Start the wave by giving the cord a quick transverse blow near one end and at the same instant start the stop watch. Count the number of returns of the wave that are clearly perceptible, and stop the watch at the last return counted. The number of returns times twice the distance between the supports divided by the time in seconds gives the velocity. The distance must be measured along the curve of the cord for each load. Use a steel tape.

The velocity of the wave is $v = \sqrt{\frac{T}{m}}$, where $T$ is the

tension of the cord in dynes and $m$ the mass per unit length of the cord. Determine the mass $m$ by weighing and measuring the entire length of the cord. For $T$ the weight supported by the cord multiplied by the acceleration of gravity may be used. From these values compute $v$ the velocity. This should check with the observed value obtained as directed above. Repeat by using a weight of approximately four kilograms.

## EXPERIMENT 115

### TO TEST EXPERIMENTALLY THE EQUATION $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

**Apparatus.**—A wire that is attached to the table at one end passes over two supports and carries a weight at the other end. At the middle point of the wire is a mechanism for determining the number of vibrations. As the wire vibrates it makes and breaks the current through an electromagnet which keeps it vibrating, and also actuates a weighted escapement that registers the number of vibrations.

**Method and Manipulation.**—A. Determine $m$ the mass of the wire per unit length, $l$ the length of the wire between the two supports, and $T$ its tension in dynes. Substitute these values in the equation, and obtain $n$, the number of vibrations of the wire per second.

B. Cause the wire to vibrate, and observe the number of revolutions made by the escapement wheel in a given time. From this determine $n$, the number of vibrations made by the wire in a second and compare this value with that obtained from the equation.

C. Repeat, using a different tension.

REFERENCE.—Sound, by Poynting & Thompson, p. 82.

## EXPERIMENT 116

### VELOCITY OF SOUND IN AIR

**Apparatus.**—A large vertical glass tube has at its upper end an ear tube *e* and a tuning fork as shown in Fig. 110. The lower end is connected to the vessels *a* and *b* by means of tubes as shown in figure. *c* and *d* are stop cocks. When *d* is closed and *c* is open the water in the resonance tube rises. When *c* is closed and *d* is open the water surface is lowered.

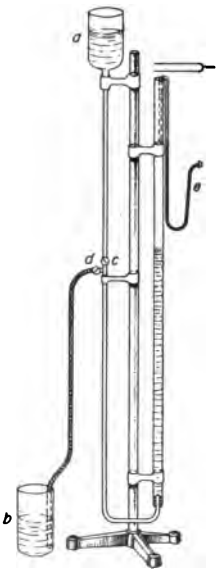


FIG. 110.

**Method and Manipulation.**—Allow the water to enter the tube. As the water rises, the reenforcement of the sound is detected through the ear tube. Place a rubber band at the position of the water surface when the reenforcement is a maximum. Repeat several times by turning the stop cocks *c* and *d*. In this way locate the position of the water surface for each point where the reenforcement of the sound is a maximum. The distance between two reenforcements is equal to a half wave length of the sound. Place bands at the highest and lowest positions noted, and measure the distance *d* between them. Repeat several times.

Let $\lambda$ be the wave length and $i$ the number of internodes in the distance $d$, then

$$\lambda = \frac{2d}{i}.$$

The velocity $V_t$ of the sound in the tube at the tem-

perature $t°$ of the room $= \lambda n$ where $n$ is the frequency of the fork. The velocity at 0° C.

$$V_0 = \frac{V_t}{\sqrt{1 + at}}.$$

The velocity $V_a$ in free air is larger than in the tube and is obtained from the equation

$$V_a = V_0\left(1 + \frac{0.007989}{2r\sqrt{\pi n}}\right). \text{(}\textit{Phil. Mag.}\text{, Sept., 1894, p. 254.)}$$

## EXPERIMENT 117

### VELOCITY OF SOUND IN A SOLID—KUNDT'S METHOD

**Apparatus.**—The solid in which the velocity is to be determined must be in the form of a long rod or tube. This is firmly clamped at its central point. A cork disc is firmly attached to one end which is placed into the end of a horizontal glass tube. The other end of the glass tube contains a movable piston.

**Method and Manipulation.**—Distribute cork dust inside the tube so as to form a thin layer throughout its length. Stroke the free half of the rod with a resined cloth, thus producing in it longitudinal vibrations. The cork disc transmits these vibrations to the air in the tube. By means of the piston at the other end, the length of the vibrating air column is adjusted until stationary waves are formed. Since compression and rarefaction take place only at the nodes, while at the internodes the air sweeps to and fro, the dust at the former remains undisturbed. This affords a means of locating the nodes and determining the wave-length in air of the sound emitted by the rod.

When a rod is clamped at its middle the wave-length, in its own substance, of the sound it emits is twice the length of the rod.

Measure the distance $d$ between the extreme nodes in the glass tube.

Let $i$ = number of internodes in length $d$,
$l$ = length of rod,
$V_t$ = velocity in air at temperature of room,
$V_s$ = velocity in solid,
$\lambda_a$ = wave-length in air,
$\lambda_s$ = wave-length in solid.

Since the period of the waves in the rod and in the air is the same

$$V_s = \frac{\lambda_s}{\lambda_a} V_t,$$

where

$$\lambda_a = \frac{2d}{i},$$

and

$$\lambda_s = 2l.$$

Use 332 meters per second as the velocity in air at zero degrees and from this determine $V_t$, as in Exp. 116.

Make several determinations of $\lambda_a$.

Substitute the values obtained and solve for $V_s$.

The difference between the velocity in free air and in a closed tube is disregarded in the above.

## EXPERIMENT 118

### FREQUENCY OF A FORK BY MEANS OF STATIONARY WAVES PRODUCED BY IT IN A CORD

**Apparatus.**—A braided silk line of about 130 cm. length and ½ mm. diameter is attached to an electric fork and supports a weight-pan as shown in Fig. 111.

**Manipulation.**—Adjust the weights until the number of loops is as large as possible. Measure the distance from pulley to the first node $a$ from the fork. From

this length obtain the value of the wave-length $\lambda$. Obtain also the mass per unit length by weighing accurately a longer length of the same line. The tension $T$ is the weight supported by the line times the acceleration of gravity. Compute the frequency $n$ from the relation

$$n = \frac{1}{\lambda}\sqrt{\frac{T}{m}}.$$

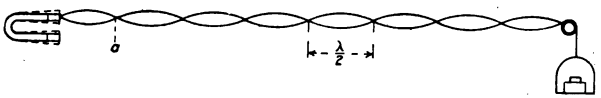



FIG. 111.

Then increase the weight until the number of loops is one less, and again determine $n$. Continue thus until the number of loops becomes as small as possible. From these results determine the average value of $n$.

## EXPERIMENT 119

### FREQUENCY OF A TUNING FORK BY THE GRAPHICAL METHOD

**Apparatus.**—The tuning fork is adjusted so that it will trace its vibrations by means of a stylus on the smoked surface of a rotating drum. A time-marker giving seconds is adjusted so that its tracing is parallel and close to the tracing of the fork.

**Manipulation.**—Cover the surface of the drum with glazed paper, and smoke the surface by holding a luminous flame against it while the drum is rotating.

Let the fork and marker make their tracings on the surface as directed above.

Determine the frequency by counting the total number of vibrations made by the fork during the total number of seconds recorded by the marker.

Also determine the number of vibrations during each interval recorded by the marker. If the mercury surface in the standard clock is not exactly in the center of the swing of the pendulum the intervals

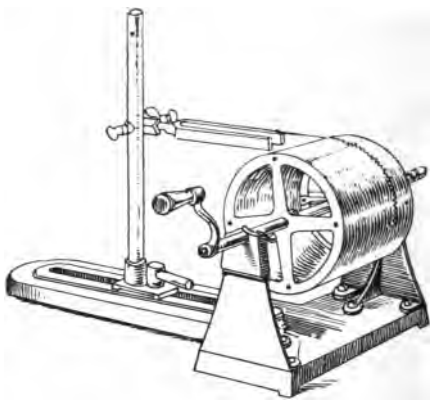


FIG. 112.

will be of two different lengths, occurring alternately. Therefore find the average number of vibrations for each series of alternate intervals, and take the mean of the two averages for the final result.

# LIGHT

## EXPERIMENT 120

### PHOTOMETRY

To investigate the illuminating power of various incandescent electric lamps by means of a Bunsen's photometer.

**Apparatus.**—A Bunsen photometer designed for the use of an electric lamp as a standard, and with a socket for the lamp to be investigated that is capable of rotation about an axis at right angles to the axis of symmetry of the lamp. The standard lamp is to be set



FIG. 113.

at such a distance from the translucent paper star that both sides of the latter appear of the same brightness. The illuminating power of the two lamps is then directly as the squares of their distances from the star.

**Method and Manipulation.**—Owing to insufficient sensitiveness of the eye the adjustment will vary according to which side of the star is originally the brighter. Hence settings should always be made in pairs, first with approaching and then with receding standard. The average reading may be used in the computations.

Make settings for computing the lateral candle power of several lamps of different rating. The lateral candle power is that on a line at right angles to the axis of the lamp.

Make settings for computing the candle power of any one lamp in directions lying in a plane through the axis of the lamp from the tip to the base. If the setting is impossible for any of the chosen directions on account of the limited motion of the standard, proceed as follows: Place a sheet of translucent glass on the side of the standard. The normal transmission coefficient of this sheet must be determined by measuring the candle power of any lamp in the same direction with and without the insertion of the glass on the side of the standard. The ratio of the true candle power to the apparently larger value when the sheet is inserted is the coefficient required. This coefficient should be determined for several different degrees of illuminating power and used in correcting the values where it has to be used.

**Calculations.**—Find the values of the candle power of the unknown lamp in the various experiments described, assuming that the candle power of the standard is known (this will be given). Plot in polar coördinates the distribution curve for the lamp whose candle power in various directions has been computed. If a translucent sheet of glass had to be used give its normal transmission coefficient (that is the fraction of the light incident normally, which is transmitted in the same direction).

## EXPERIMENT 121

### SPHERICAL MIRRORS

**Apparatus.**—The spherical mirror $M$ is mounted so that it may be rotated about a vertical axis, also so that it may be moved along the metric scale that supports it. A vertical rod $o$ serves as the object. Back of the rod $o$ and mounted on the same slide is a plane mirror $N$ placed vertically and at an angle of forty-five degrees with the supporting scale. The purpose of this mirror is to produce a bright background against which

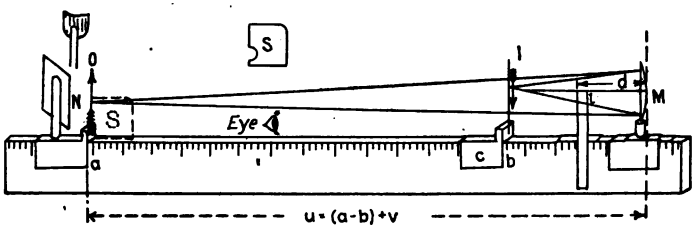



FIG. 114.

the image may be clearly seen. The image $I$ is located by adjusting a vertical rod $c$ until there is no parallax between it and the image. The position of the mirror is determined by placing the end of the rod $l$ against the surface of the mirror and obtaining the reading on the scale opposite the vertical edge attached to $l$ and adding to this reading the length $d$ obtained by means of a vernier caliper. The readings $a$ and $b$ for the position of the object and image on the scale are obtained by means of the square $s$.

#### A—REAL IMAGE

**Method.**—In the general equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R} = \text{a constant},$$

$u$ is the distance of the object from the mirror, $v$ the distance of the image, $f$ the focal length, and $R$ the radius of curvature of the mirror. Determine $v$ for each of several values of $u$ and substitute in succession the values of each set in the general equation. Solve for $f$ in each case and average. The equation is verified if the several values obtained for $f$ are constant.

### B—Virtual Image

**Apparatus.**—Same as in A except that the concave or convex mirror has a small hole at center or has a small area from which the silvering has been removed.

**Method.**—To obtain the distance of the virtual image place the rod $c$ behind the mirror, and adjust until there is no parallax between the image and the rod as seen through the hole or unsilvered area.

Verify the equation for each of the two mirrors, as in case A.

## EXPERIMENT 122

### TO MEASURE THE FOCAL LENGTHS AND RADII OF CURVATURE OF SPHERICAL MIRRORS

**Method.**—Find the distance of the image from each of the two kinds of mirrors used in Exp. 121, when the rays coming from the object are nearly parallel. For this purpose use an object situated at a long distance. Then in each case $f = v$, and $R = 2f$.

Also find $R$ by moving the mirror towards the object until $u = v$., *i.e.*, until there is no parallax between the object and the image. Then $u = v = R$.

The values thus obtained for $f$ and $R$ should check with the values obtained in $A$ and $B$ of the previous experiment.

## EXPERIMENT 123

### REFRACTIVE INDEX OF GLASS—APPROXIMATE METHOD

**Apparatus.**—A spectrometer consisting of the following parts: a force-table graduated in degrees or half-degrees, a collimating tube with attached screen to cut off extraneous light, a prism table capable of rotation about the vertical axis of the force-table, a



FIG. 115.

sighting tube with peep-sight. A Bunsen burner with a bead of NaCl supported so as to touch the side of the flame.

**Method and Manipulation.**—Clamp the collimating tube so that its axis points toward the center of the force-table (the point on top of the prism clamp is directly over the center) and place the sodium flame so that light passing through the collimator will strike one face of the prism. Place the sighting tube so that the image of the collimating tube formed by the prism will appear in line with the center of the disc when viewed through the sighting tube. Find by trial the position of the prism and sighting tube for minimum deviation (the faces of the prism will then be equally inclined to the axes of the collimating and sighting

tubes respectively). When this adjustment has been correctly made a slight rotation of the prism in *either* direction will cause the image of the collimating tube to move in such a direction that the sighting tube would have to be moved toward the collimating tube to restore the alignment through the center of the force-table. Record the positions on the circular scale of both the collimating and sighting tubes. Readjust for minimum deviation on the other side of the direction of the incident light and record the new position of the sighting tube. Repeat several times for different positions of the collimating tube so that the same scale readings will not recur.

**Calculations.**—Compute the average angle of minimum deviation from all the readings taken. Calling this average value $D$ and the angle at the vertex of the prism $A$ (this is 60.0°, to the accuracy indicated, for all the prisms supplied), the required refractive index for the yellow lines of the sodium spectrum, which supply practically all the light used in making the settings, is given by the formula

$$\mu = \frac{\sin (A + D)/2}{\sin A/2},$$

Compute the value of $\mu$.

For a more accurate method see Exp. 130.

## EXPERIMENT 124

### LENSES

**Apparatus.**—A convex lens; a half of a convex lens a half of a concave lens. Apparatus as shown in Fig. 116 In this case the reading on the scale for the midway point of the thickness of the lens must be determined. This is done by placing the end of the rod $l$ against the

surface of the lens and obtaining the reading on the scale for the vertical edge, adding or subtracting the distance $d$ and one-half the thickness of the lens as determined by means of a vernier caliper.

Determine the scale readings for the object $a$ and for the image-finder $c$ by means of the square.

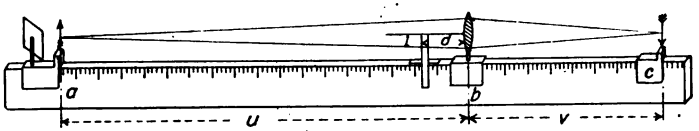



FIG. 116.

## A.—REAL IMAGE

In the general equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \text{a constant},$$

$u$ is the distance of the object from the center of the lens, $v$ the distance of the image, and $f$ the focal length of the lens. Distances on the same side of the lens as the object are positive.

Thus $u$ is positive, $v$ is negative for a real image, and $f$ must therefore be negative for a converging lens.

Determine $v$, by parallax method, for each of several values of $u$ and solve for $f$ and average. The settings can be made more accurately if the image-finder is made to cross the image at a very acute angle.

## B.—VIRTUAL IMAGE

*a.* Place the object in front of a half of a double convex lens in a position where a virtual image is obtained. Move the rod $c$ behind the lens and observe it over the cut edge of the lens, so that the rays from it do not pass through the lens. The place where no parallax exists

between the image of the object and the rod *c* is the location of a virtual image. Verify equation as in A.

*b.* Use a half of a double concave lens, locate the position of the virtual image by the parallax method, and verify the general equation as in A above.

## EXPERIMENT 125

### TO MEASURE THE FOCAL LENGTH OF LENSES—ALTERNATIVE METHOD

**Method.**—Find by the parallax method the distance of the image of a distant object for each of the two kinds of lenses used in Exp. 124. This requires the determination of the scale readings corresponding to the middle point of the thickness of the lens and to the image-finder in each case. Make several settings, using different objects all more than 30 meters away. This gives directly the focal lengths of the lenses, and the values obtained should check with those obtained in Exp. 124.

## EXPERIMENT 126

### THE MAGNIFYING POWER OF OPTICAL INSTRUMENTS

From a point upon an object, the eye receives a cone of rays the angle of which becomes greater as the distance between the eye and object becomes less. The point is clearly seen when the eye is able to bring this cone of rays to a focus upon the retina. The distance at which the point is seen most clearly is in the case of most eyes about 25 cm. If the object is within this distance it can still be seen distinctly by the aid of a lens which converges the rays so as to make them appear to come from a greater distance which by proper adjustment

of the lens can be made that of the most distinct vision (25 cm.). This lens, or simple microscope is then said to be focused for the object.

The angle between the axes of the two cones of rays received by the eye from two extreme points of the object or image determines the apparent size of the object or image.

The magnifying power of any optical system is defined as the ratio of the apparent size of the final image when viewed by the eye at the most favorable distance (25 cm.) to the apparent size of the object when most favorably placed for observation by the naked eye.

In the case of microscopes of all kinds the object is considered to be most favorably placed when at 25 cm. from the eye.

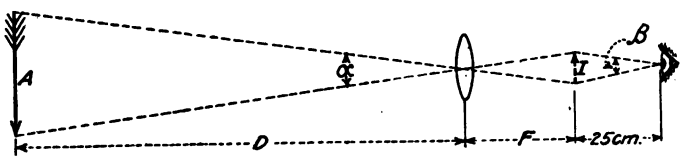



FIG. 117.

In the case of a single lens or mirror forming a real image and in the case of a telescope the object is considered to be most favorably placed when at its actual distance from the eye.

**A. Magnifying Power of a Convex Lens when the Object Viewed is at a Great Distance.**—The image in this case is formed at practically the principal focus of the lens, and the distance of the object from the lens, $D$ in Fig. 117, can be considered as practically the distance of the object from the eye. In observing the image $I$, the eye is placed 25 cm. from it, i.e., at the distance of most distinct vision. The angle formed at the eye by the image $I$, is approxi-

mately equal to $\frac{I}{25}$ and the angle formed by the object itself is

$$\frac{A}{D+F+25} \approx \frac{A}{D} = \frac{I}{F}.$$

Therefore the magnifying power $M = \frac{\beta}{\alpha} = \frac{I}{25} \div \frac{I}{F} = \frac{F}{25}$.

**Manipulation.**—Place a scale horizontally at a distance of 25 cm. from the eye. Adjust the lens so that there is no parallax between this scale and the image $I$ of the distant object.

1. Determine the distance $F$ and compute the magnifying power $M$.

2. With the eye 25 cm. from $I$, measure the distance between two points on the image by means of the scale at $I$. Remove the lens, and with the eye and scale in the same position, measure the distance between the same two points on the object directly. The ratio of these distances should check with the computed value of $M$.

**B. Magnifying Power of a Convex Lens when the Object Viewed is within the Principal Focus. (Simple Microscope.)**—The image produced is a virtual image and the microscope is said to be in focus for the object when the image is seen most distinctly, i.e., when it is at the distance of most distinct vision or about 25 cm. from the eye. The object observed is nearer the eye than the image and for comparison of magnification must be assumed to be placed 25 cm. from the eye or at the same distance as the image. The eye is assumed to be placed very *near the*

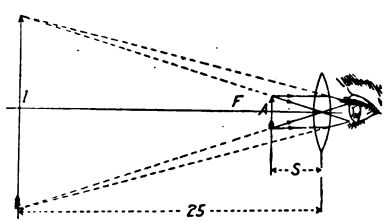



FIG. 118.

*lens* so that the angle formed at the center of the lens is nearly the same as that formed at the eye.

Then

$$M = \frac{I}{25} \div \frac{A}{25} = \frac{I}{A} = \frac{25}{S}.$$

**Manipulation.**—Focus the lens on a vertical graduated scale $A$, and place another similar scale 25 cm. from the lens. With one eye, observe the image of the first scale, and with the other, view the second scale directly. Adjust the distance $S$ by moving $A$ until the image of the first scale appears beside the second scale and has no parallax with it.

Move the eyes vertically in testing this adjustment since their accommodation for different distances permits independent rotations about vertical axes, causing apparent lateral shift.

1. Determine the value of $S$ and calculate the magnifying power of the lens from the equation

$$M = \frac{25}{S}.$$

2. With one eye observe the image of the first scale and with the other eye view the second scale directly. Determine the number of scale divisions on one that corresponds to a given number on the other. The ratio of these is the magnification and should check with the computed value.

**C. Magnifying Power of a Compound Microscope.**—The object $A$ is a small distance beyond the focus of the objective $N$, so as to form a real image $I_1$ at some distance from the objective. This image is within the principal focus of the eye-piece $X$ which acts as a simple microscope.

The real image is larger than the object, its magnification being

$$M_1 = \frac{I_1}{A} = \frac{L}{R}.$$

The magnification of this image by the eye-piece is

$$M_2 = \frac{25}{S}.$$

The total magnification of the object is

$$M = M_1 M_2 = \frac{25L}{RS}.$$

This assumes the eye to be very near the lens.

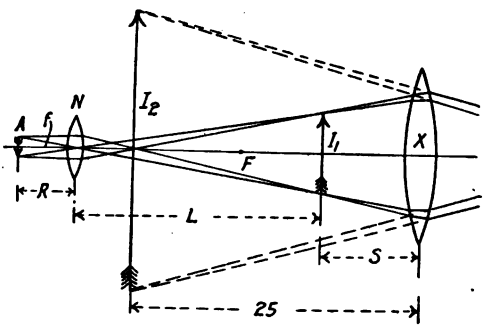



FIG. 119.

**Manipulation.**—Determine the distance $S$ for the lens $X$, to be used as the eye-piece, as directed under manipulation in case B above. Having done this leave the scale at $I_1$ in position and remove the lens $X$. Place a vertical scale $A$ and the lens $N$, which is the objective, so that a real image of $A$ is formed. Then adjust $A$ and $N$ until there is no parallax between the real image of $A$ and the scale at $I_1$. Replace the lens $X$. The final image $I_2$ of A is then seen at a distance of 25 cm. from $X$, the eye being close to the lens $X$.

1. Measure the distance $R$, $L$ and $S$, and compute the magnification $M$.

2. With one eye, observe the image $I_2$ through the lens, and with the other eye, observe directly a scale placed at 25 cm. from the eye, either on the same support as the other parts or held at one side of it. The ratio of the number of divisions on $I_2$ that correspond to a given number on the scale at 25 cm. is the magnification, and should check with the computed value of $M$.

**D. Magnifying Power of an Astronomical Telescope.**—The object $A$, Fig. 120, is assumed to be at a considerable distance from the lens $N$. The magnifying power of the objective as shown in section A is $M_1 = \frac{F}{25}$.

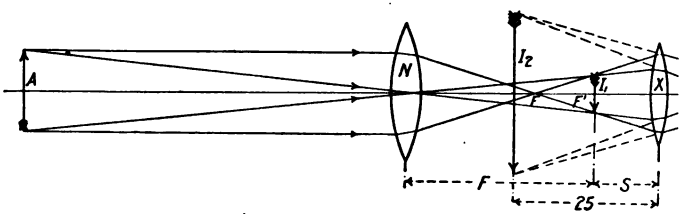



FIG. 120.

When the image falls within the principal focus of the eye-piece $X$, the magnification of this image by the eye-piece is $M_2 = \frac{25}{S}$, as shown in section B.

The total magnifying power $M = M_1 M_2 = \frac{F}{S}$.

**Manipulation.**—The procedure is the same as under manipulation in section C, except that the lenses $X$ and $N$ are interchanged, and a distant object is used instead of the scale at $A$.

1. Determine the distances $F$ and $S$ and compute the magnification $M$.

2. Hold a scale at a distance of 25 cm. from the eye-

piece. On this scale measure the distance between two points on the image by viewing the image with one eye and the scale with the other. Then with the eye and scale in the same position, measure on the scale directly the distance between the same two points on the object. The ratio should check with the computed value for $M$.

**E. Terrestrial Telescope.**—Place the reversing lenses between the objective and the eye lens and at a distance

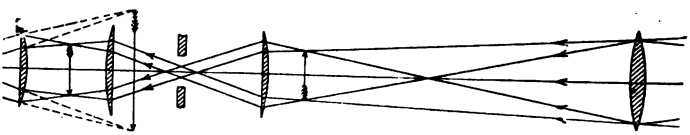


FIG. 121.

from the real image equal to the focal length of the first reversing lens. Since the two lenses are alike, the second image will be formed at a distance equal to the same focal length. Place the eye lens at a distance $S$ from the second image. Observe the erect image.

The magnifying power is not affected by this use of the reversing lenses.

## EXPERIMENT 127

### ADJUSTMENT OF THE SPECTROMETER

The spectrometer consists of a circular graduated disc mounted horizontally on a vertical axis supported by a leveling base. Two arms extend horizontally from the axis. One supports a telescope and the other a collimator. The arm supporting the telescope may be unclamped from the vertical axis and rotated so as to make any desired angle with the collimator. Beneath the telescope is a tangent screw for moving the telescope slowly. Surrounding the graduated disc is a rim

which moves with the telescope and on which are two verniers placed 180° apart.

The telescope is made of three tubes. The first is the eye-piece and contains a thin glass plate set at an angle of 45° with the axis of the telescope and directly opposite a round opening in the side of the eye-piece. The second contains the cross-hairs, and is moved in the third by means of a rack and pinion. The third

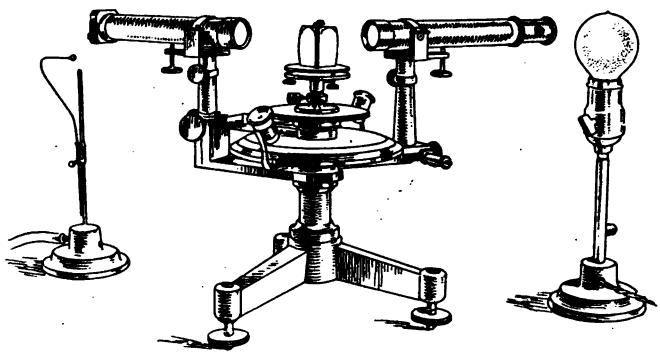


FIG. 122.

contains the objective, and is fixed to the supporting arm. The collimator consists of two tubes. One is fixed to the arm, and contains a lens similar to the objective of the telescope. The other slides in the first and contains an adjustable slit.

**1. Focus the Eye-piece on the Cross-hairs.**—This is done by moving the eye-piece in or out until the cross-hairs are seen distinctly.

**2. Focus the Telescope for Parallel Rays.**—In the experiments to follow it is necessary that rays coming from the collimator be parallel and hence the telescope must be adjusted for parallel rays. This may be done by focusing the telescope on a distant object. The following is a more exact method and is the one to be used.

Place a prism at the center of the spectrometer disc and adjust by means of the screws underneath until one face is about perpendicular to the axis of the telescope. Place a light at the side of the eye-piece so that the rays enter the opening as shown in Fig. 122. The glass plate reflects a portion of the rays down the barrel of the telescope. In passing the cross-hairs some of the rays are cut off. The remaining rays pass through the objective to the face of the prism where a portion is again reflected. Since the face of the prism was placed nearly perpendicular to the axis of the telescope a portion of the reflected rays will again enter the telescope. If the cross-hairs are in the principal focus, rays proceeding from them are parallel while passing from the objective to the prism. In this case, the rays reflected by the prism will be parallel, and on entering the telescope, will be brought to a focus in the plane of the cross-hairs forming an image of them which may be seen through the eye-piece. Therefore, when the telescope is adjusted for parallel rays the cross-hairs and their image are in the same plane and no parallax is observed when the eye is moved in front of the eye-piece.

**3. Adjust the Collimator for Parallel Rays.**—This is done by turning the adjusted telescope into line with the collimator and moving the collimator slit in or out until it is seen through the telescope clearly and without parallax.

**4. Alignment.**—Align the telescope and collimator with the center of the spectrometer disc. This is done by unclamping the telescope or the collimator from its supporting arm, sighting along its barrel and turning until it points to the center of the disc.

**5. Level the Disc of the Spectrometer.**—Place a level on the disc so that it is parallel to a line through two of the screws in the base, and level. Then place the level

at right-angles to its first position, and level by turning the third screw.

**6. Level the Telescope and the Collimator.**—Place the level on top of the barrel of the telescope, and adjust by turning the vertical screw underneath. In a similar manner level the collimator.

**7. To Adjust the Face of a Prism so that it is Perpendicular to the Axis of the Telescope.**—When the face is perpendicular, rays coming from the objective to the prism parallel to the axis of the telescope will be reflected back in their paths. In this case, since the intersection of the cross-hairs is in the principal focus, rays coming from the intersection will retrace their paths and hence come to a focus at the intersection. The superposition of the cross-hairs and their image is therefore the test for perpendicularity.

**Eccentricity.**—If the rim that carries the verniers does not rotate about the center of the graduated disc, the zero line on the vernier will, for the same degree of rotation, move over an unequal number of divisions on the disc for different portions of the circumference and hence give rise to an error in angular measurements. This error is partly eliminated by measuring the angle with each of the two attached verniers placed 180° apart and taking the average.

## EXPERIMENT 128

### ANGLE OF INCIDENCE IS EQUAL TO THE ANGLE OF REFLECTION

**Apparatus.**—A spectrometer and a glass prism.

**Method and Manipulation.**—Adjust the spectrometer according to the directions given in Exp. 127. Place the prism as indicated in Fig. 123. Move the telescope into the position $N$ and adjust the face of the prism so that

it is perpendicular to the axis of the telescope. Obtain the reading $N$ for this position. Then turn the telescope into the position $R$ so that the intersection of the cross-hairs coincides with the center of the reflected slit. Obtain the reading $R$ for this position. Then

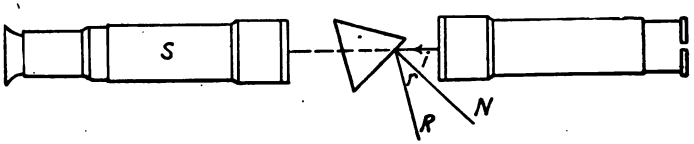



FIG. 123.

remove the prism, and turn the telescope into line with the collimator, and obtain the reading $S$ for this position.

$$i = 180 - (S - N),$$

and

$$r = R - N,$$

then if

$$i = r,$$

$$R - N = 180 - (S - N).$$

## EXPERIMENT 129

### ANGLE OF A PRISM

**Apparatus.**—A spectrometer and a prism.

**Method and Manipulation.**—Adjust the spectrometer. Since the angle between normals to two faces of the prism has to be measured, it is necessary that the prism be perpendicular so that the cross-hairs and their reflected image will coincide for each face. Adjust one face so that it is perpendicular, then bring the telescope to the other and adjust so that the cross-hairs and their image coincide. It is here essential not to disturb

the perpendicularity of the face previously adjusted. This is avoided by turning the proper screw. The prism is placed on the plate that supports it in such a way that each face is perpendicular to a line through two of the supporting screws. (See Fig. 124.) If the face *de* has been made perpendicular, the face *df* may be adjusted without disturbing *de* by turning the screw *a* as it will be moved only in its own plane. After the second face has been adjusted, turn the telescope back to the first, and determine if its adjustment has been disturbed. If so, correct, and then again test the second.

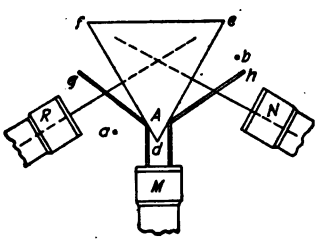


FIG. 124.

When the prism has been made perpendicular superpose the cross-hairs and their image and record the reading for the normal *R*. Then turn the telescope to the other face and record the reading for the normal *N*. The angle of the prism is

$$A = 180 - (R - N).$$

It is easier here to leave the telescope and lamp fixed, turning the prism table clamped to the graduated circle, and using, if necessary, the tangent screw under the collimator.

A second method for obtaining the angle of the prism is to place the collimator at *M*, Fig. 124, and measure the angle between the reflected rays *g* and *h*. This angle is twice the angle of the prism.

It is necessary here to turn the telescope, and the appropriate tangent screw is the one beneath the telescope. In either method keep the readings on the two verniers separate; compute the angle of the prism from each set of readings independently; and

finally take the mean of the two values so obtained. Designate the verniers as to the right or left of the telescope, since they always have the same positions relative to it.

## EXPERIMENT 130

### REFRACTIVE INDEX OF GLASS FOR SODIUM LIGHT

**Apparatus.**—A spectrometer and the glass prism whose angle was determined in Exp. 129, a Bunsen burner and a bead of NaCl on a wire.

**Method and Manipulation.**—Adjust the spectrometer as before, and then adjust the prism so that its reflecting faces are perpendicular to the disc of the spectrometer. To measure the angle of minimum deviation, turn the prism into the position indicated in Fig. 125 and allow sodium light to pass from the collimator through the prism and into the telescope on the other side. Turn the prism and telescope until the refracted slit is seen. It will be found on turning the prism in one direction that the image of the slit moves toward the axis of the collimator produced, stops, and then returns. The turning point, therefore, marks the direction of the beam when it is deviated the least and the angle between this and the original direction is the angle of minimum deviation to be determined. Place the intersection of the cross-hairs on the image of the slit when the latter is at the turning point. Record the reading $B$ for this position. Then remove the prism, and turn the telescope into line with the collimator and record the reading $S$.

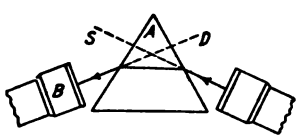



FIG. 125.

The angle of minimum deviation $D = S - B$, and the refractive index

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A},$$

where $A$ is the angle of the prism.

## EXPERIMENT 131

### WAVE LENGTH OF LIGHT—FIRST DIFFRACTION GRATING METHOD

**Apparatus.**—A diffraction grating; a scale with a slit (*a*) Fig. 126; a Bunsen flame; a bead of NaCl for the production of the sodium light. The apparatus is arranged as shown in Fig. 126. In this figure *a* is the slit, *G* is the grating. Images of the first three orders are

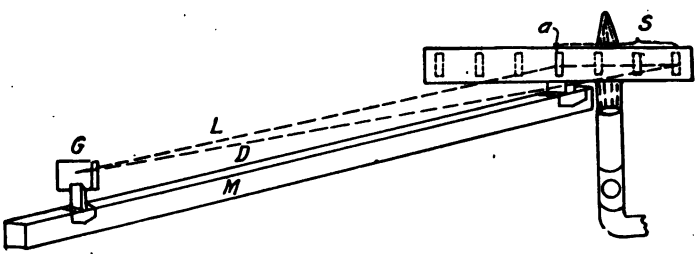



FIG. 126.

shown on each side of the slit, but can, of course, only be seen through the grating. Fig. 126 is a plan of the apparatus showing the notation used below. $P$ is the distance from the slit to the first image, and $S$ is the distance to the $n$th image. A sliding pointer is used to facilitate reading the scale on the screen.

**Method.**—The diffracted images of the slit are observed through the diffraction grating $G$ placed at a distance of about a meter. Get the scale reading on the

screen for the center of each image that is visible when the flame is freshly supplied with NaCl. Depending on the grating space, which can be computed from the number of grating spaces per millimeter (to be given in each case), one, two, three, or more images can be seen on each side of the slit by darkening the room sufficiently.

Determine the distance from the image of the highest order on one side to the image of corresponding order on the other side. One-half of this distance is $S$ the distance to the slit. If the grating used is large, care must be

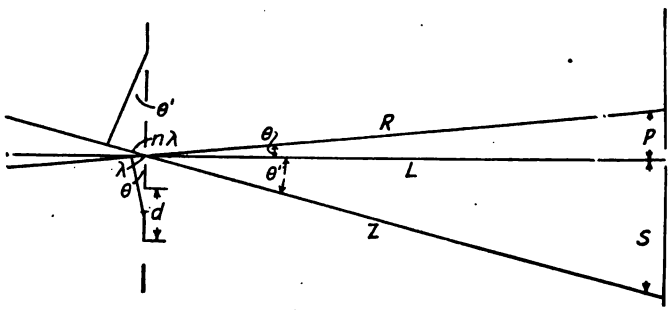



FIG. 127.

taken to have the eye near the grating and at a point directly above the scale $M$. Or, more generally, place the eye so that the line of sight passes through the center of the grating. The rays coming from the slit can be assumed to be nearly parallel, and, if the first diffracted image is observed,

$$\lambda = d \sin \theta, \text{ (Fig. 127),}$$

but

$$\sin \theta = \frac{P}{R},$$

$$\therefore \lambda = d \frac{P}{R}.$$

In case the $n$th image is observed,

$$n\lambda = d \sin \theta,$$

and

$$\lambda = \frac{d}{n} \frac{S}{Z}.$$

Find the average wave length for the sodium light, using various values of $L$.

## EXPERIMENT 132

### WAVE LENGTH OF LIGHT—SECOND DIFFRACTION GRATING METHOD

**Apparatus.**—A spectrometer and a diffraction grating.

**Method and Manipulation.**—Adjust the spectrometer. Then place the grating perpendicular to the collimator. This may be done in the following manner: Turn the telescope into line with the collimator; and place the cross-hairs on the image of the slit; place the grating on the stand between the telescope and the collimator, and adjust by rotating and tilting the grating until it is perpendicular to the axis of the telescope. Then as the telescope and the collimator are in line, the grating is also perpendicular to the collimator. Allow sodium light to pass through the slit. Turn the telescope to one side until the first diffracted image of the slit enters the field of the telescope. Place the cross-hairs on the image, and record the reading $N$. Turn the telescope to the first diffracted image on the other side, and obtain the reading $M$.

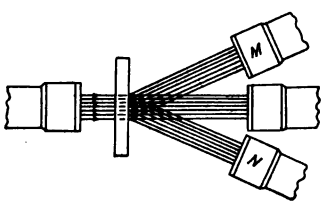



FIG. 128.

If $n$ = the number of lines per millimeter on the grating,
$\theta$ = the angle of diffraction,
$\lambda$ = the wave length,
$d$ = the distance between two adjacent lines,

then

$$\theta = \frac{N - M}{2},$$

and

$$\lambda = d \sin \theta.$$

## EXPERIMENT 133

### POLARIZED LIGHT

**Apparatus.**—A polariscope; a uniaxial crystal cut parallel to its optic axis (selenite or mica); a uniaxial crystal cut perpendicular to its optic axis (Iceland spar);

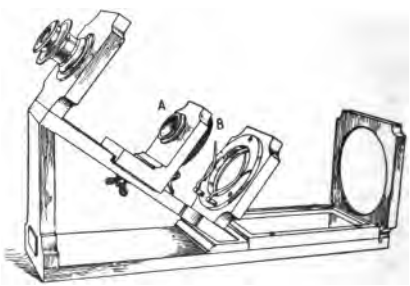



FIG. 129.

a biaxial crystal cut perpendicular to a line bisecting the angle between the axes (niter crystal); a quartz crystal cut perpendicularly to its optic axis.

The polariscope consists of a polarizer made of a number of glass plates and placed in a horizontal position. At one end of the polarizer is a vertical screen of ground glass for diffusing the light passed through it to the

polarizer. From the polarizer, the light passes to a large lens serving as a condenser near the focus of which is placed a smaller lens which renders the rays parallel. The light then enters the analyzer consisting of a Nicol's prism.

**Manipulation.**—Assume the light waves corming from the polarizer to be vibrating in a plane peependicular to the plane containing the incident and reflcted rays.

The principal section of the Nicol's prism coincides with the plane of vibration of the light that it transmits. Determine its direction with reference to the angles in the end view of the prism.

**Uniaxial Crystal Cut Parallel to the Optic Axis.**

1. Turn the analyzer until its principal section is horizontal and place a one-half wave plate of selenite or mica crystal in the parallel rays, i.e., position *A* Fig. 129. Since the crystal is cut parallel to the optic axis it has two planes of vibration that transmit polarized white light. Rotate the crystal, and determine their direction with reference to the frame of the crystal.

2. Use sodium light instead of white, and rotate the crystal until its planes of vibration make an angle of 45° with the vertical. Rotate the analyzer, and observe the positions for which light and darkness are obtained. Explain.

3. Replace the white light and rotate the crystal until its planes of vibration make an angle of 45° with the vertical. Rotate the analyzer, and observe if two colors are obtained and if they are complementary. Explain.

4. Rotate the crystal, and observe if more than one color is obtained. Explain.

5. Remove the one-half wave plate, and insert in its place a quarter wave plate. Rotate the analyzer and observe if any change takes place when sodium light is used. Explain.

6. Repeat using white light. Explain.

**Uniaxial Crystals Cut Perpendicularly to the Optic Axis.**

1. Place the principal section of the analyzer vertical and insert the crystal of Iceland spar in the converging rays of white light, i.e., position *B*. Observe the black cross. Explain.

2. Between the arms of the cross observe the colored rings. Explain.

3. Rotate the crystal and observe if any change takes place. Explain.

4. Rotate the analyzer 90°. Observe the white cross and at any point between the arms observe that the color is complementary to that seen in position 2. Explain.

**Biaxial Crystal Cut Perpendicularly to the Bisector of the Angle Between the Optic Axes.**

1. Place the principal section of the analyzer vertical. Place the biaxial (niter) crystal in position *B*. Rotate the crystal until a line joining the centers of the two observed circles makes an angle of 45° with the vertical. Observe the hyperbola and the colored bands between its arms. Explain.

2. Rotate the analyzer 90°. Observe the white hyperbola and at intervening points note that the color is complementary to that observed in position 2. Explain.

3. Rotate the crystal 45° and observe the white cross. Then rotate the analyzer 90° and observe the black cross. Explain.

**Quartz Crystal Cut Perpendicularly to the Optic Axis.**

1. Rotate the analyzer until its principal section is horizontal. Place the quartz crystal in position *A* and use a sodium flame as the source of light. Rotate the analyzer until the light becomes a maximum. Observe the angle of rotation.

2. Replace the sodium flame by lights of different colors, and measure in each case the angle of rotation.

3. Use white light, and observe the different colors obtained by rotating the analyzer. Explain.

4. Place the crystal at $B$, and observe the rings. Explain.

# APPENDIX

## 1. LENGTH

| | | | |
|---|---|---|---|
| 1 centimeter | = .39370 inch | 1 micron ($\mu$) = | .001 mm. |
| 1 meter | = 3.2809 feet | 1 millimicron ($\mu\mu$ or m$\mu$) = | .001 $\mu$ |
| 1 kilometer | = 0.62137 mile | 1 Angström unit | $= 10^{-8}$ cm. $= .1\ \mu\mu$ |
| 1 mil | $\triangleq$ .001 in. | | |
| 1 inch | = 2.54 cm. | | |
| 1 foot | = 30.46 cm. | | |
| 1 mile | = 1.609 km. | | |

## 2. AREA

| | | | |
|---|---|---|---|
| 1 sq.cm. | = 0.155 sq.in. | 1 sq.in. | = 6.4516 sq.cm. |
| 1 sq.m. | = 10.764 sq.ft. | 1 sq.ft. | = 929.03 sq.cm. |
| 1 circular cm. | = 0.1217 sq.in. | 1 sq. mile | = 2.59 sq.km. |
| area of circle | $= \left(\frac{\pi}{4}d^2\right) = .7854d^2$ | 1 circular in. | = 5.0671 sq.cm. |
| | | 1 circular mil | = 0.00050671 sq.mm. |
| 1 circular mil | = 0.000001 sq.in. | | |

## 3. VOLUME

| | | | |
|---|---|---|---|
| 1 cu.cm. | = 0.061 cu.in. | 1 cu.in. | = 16.387 cu.cm. |
| 1 cu.m. | = 35.315 cu.ft. | 1 cu.ft. | = 0.028317 cu.m. |
| 1 liter | = 61.023 cu.in. | 1 cu.foot | = 28.317 liters |
| 1 liter | = 2.11336 pints (liquid) | 1 pint (liquid) | = 0.47318 liter |

## 4. MASS

| | | | |
|---|---|---|---|
| 1 gram | = 15.432 grains | 1 ounce (av.) | = 28.35 grams |
| 1 kilogram | = 2.205 pounds | 1 pound (av.) | = 453.59 grams |

## 5. CIRCULAR MEASURE

| | | | |
|---|---|---|---|
| 1 radian | = 57.296 degrees | 1 degree | = 0.017453 radian |
| $\pi$ | = 3.1416 | Log $\pi$ | = .49714987 |
| $\frac{1}{\pi}$ | = 0.31831 | | |
| $\pi^2$ | = 9.8696 | | |
| $\sqrt{\pi}$ | = 1.77245 | | |

## 6. FORCE

| | | | |
|---|---|---|---|
| 1 dyne | = 0.00007233 poundal | 1 poundal | = 13825 dynes |
| 1 dyne | = 0.00102 gram | 1 poundal | = 0.03108 pound |
| 1 dyne | $= 22.48 \times 10^{-7}$ pounds | 1 poundal | = 14.10 grams |
| 1 megadyne | = 1000000 dynes | | |

## 7. PRESSURE

| | | | |
|---|---|---|---|
| 1 atmosphere (standard) | =76.0 cm. | 1 atmosphere (76 cm.) | =29.921 in. |
| 1 atmosphere | =1.0333 kg. per sq.cm. | 1 atmosphere | =14.697 lb. per sq.in. |
| 1 atmosphere | =1.0133 megadynes per sq. cm. | 1 atmosphere | =33.90 ft. of water |
| 1 dyne per sq. cm. | =0.0004666 poundal per sq.in. | 1 poundal per sq.in | =2142.95 dynes per sq.cm. |
| 1 gram wt. per sq.cm. | =0.014223 lb. per sq.in. | 1 pound per sq.in. | =70.31 grams per sq.cm. |
| 1 cm. of mercury (0° C.) | =13.596 g.wt. per sq.cm. | 1 in. of mercury (0° C.) | =34.533 g.wt. per sq.cm. |
| 1 cm. of mercury (0° C.) | =0.19338 lb. wt. per sq.in. | 1 in. of mercury (0° C.) | =0.4912 lb. wt. per sq.in. |
| 1 megabarie | =1 megadyne per sq.cm. | | |
| 1 megabarie | =75.0068 cm. of Hg. | | |

## 8. ENERGY

| | | | |
|---|---|---|---|
| 1 erg | $=2.373\times10^{-6}$ ft.-poundal | 1 ft.-poundal | =421,390 ergs |
| 1 erg | $=7.376\times10^{-8}$ ft.-pound | 1 ft.-pound | =1.35573 joules |
| 1 g.cm. | $=7.233\times10^{-5}$ ft.-pounds | 1 ft.-pound | =13825.5 g.cm. |
| 1 joule | $=10^{7}$ ergs | | |

## 9. POWER

| | | | |
|---|---|---|---|
| 1 watt | $=10^{7}$ ergs per sec. | 1 ft.-poundal per sec. | =421390 ergs per sec. |
| 1 watt | =23.731 ft.-poundals per sec. | 1 ft.-pound per sec. | =1.35573watts |
| 1 watt | =0.7376 ft.-lb. per sec. | 1 horse-power sec. | =745.65 watts |
| 1 watt | =0.001341 horse-power-sec. | 1 horse-power sec. | =550 ft.-pounds. |
| 1 kilowatt-hr. | =2,655,403 ft.-pounds. | 1 horse-power sec. | =178.122 calories |
| 1 kilowatt-hr. | =1,3411 horse-power-hour | | |
| 1 kilowatt-hr. | =859,975 calories | | |

## 10. HEAT

| | |
|---|---|
| 1 calorie (g.c.) = .0039683 B.T.U. | 1 B.T.U. = 252.00 calories |
| 1 calorie " = 4.1862 joules | 1 B.T.U. = 1055 joules |
| 1 calorie " = 3.088 ft.-lbs. | 1 B.T.U. = 778.1 ft.-lbs. |
| 1 calorie " = 0.005614 horse-power-sec. | 1 B.T.U. = 1.4147 horse-power-sec. |

## 11. TIME

1 sidereal sec. = .99727 sec. (mean solar).
1 sec. (mean solar) = 1.002738 sidereal sec.
Length of seconds pendulum latitude 45° = 99.3555 cm. = 39.1163 in.

## 12. LOGARITHM

| | | |
|---|---|---|
| $\log_{10} N$ | $= .43429 \log_e N$ | $\log_e N = 2.3026 \log_{10} N.$ |
| $e$ | $= 2.7183$ | |

## 13. DATA WITH REGARD TO EARTH

| | |
|---|---|
| Force of gravity (Minneapolis $\phi =$ 44° 58′.7, $\lambda = 93°$ 13′.9, $h = 256$ meters) | = 980.596 dynes |
| Gravitation constant | $= 666.07 \times 10^{-10}$ |
| Mean density of the earth | = 5.52470 |
| Half diameter of earth (equatorial) | = 6378.2 km. |
| Half diameter of earth (polar). | = 6356.5 km. |
| Half diameter of earth (average) | = 6367.4 km. |
| Half diameter of earth (average) | = 3956.5 miles |
| Mean distance from earth to sun | $= 9.29 \times 10^7$ miles |
| Mean distance from earth to moon | $= 2.39 \times 10^5$ miles |
| 1° latitude at 40° latitude | = 69.0 miles |

## 14. VELOCITY

| | | |
|---|---|---|
| Velocity of light per sec. | = 186330 miles | = 299870 kilometers |
| Velocity of sound per sec. (air 0° C.) | = 1089 ft. | = 331.92 meters |
| Velocity of sound per sec. (brass) | = 11480 ft. | = 3500 meters |
| Velocity of sound per sec. (cast steel) | = 16360 ft. | = 4990 meters |
| Velocity of sound per sec. (water) | = 4728 ft. | = 1441 meters |

## 15. NUMERICAL VALUES OF THE FACTOR $\frac{0.6745}{\sqrt{n(n-1)}}$ *

| n | $\frac{0.6745}{\sqrt{n(n-1)}}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | ..... | ..... | 0.4769 | 0.2754 | 0.1947 | 0.1508 | 0.1231 | 0.1041 | 0.0901 | 0.0795 |
| 1 | 0.0711 | 0.0643 | .0587 | .0540 | .0500 | .0465 | .0435 | .0409 | .0386 | .0365 |
| 2 | .0346 | .0329 | .0314 | .0300 | .0287 | .0275 | .0265 | .0255 | .0245 | .0237 |

*For use with $\Sigma v^2$.

## 16. NUMERICAL VALUES OF THE FACTOR $\frac{0.8453}{n\sqrt{n-1}}$ *

| n | $\frac{0.8453}{n\sqrt{n-1}}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | ..... | ..... | 0.4227 | 0.1993 | 0.1220 | 0.0845 | 0.0630 | 0.0493 | 0.0399 | 0.0332 |
| 1 | 0.0282 | 0.0243 | .0212 | .0188 | .0167 | .0151 | .0136 | .0124 | .0114 | .0105 |
| 2 | .0097 | .0090 | .0084 | .0078 | .0073 | .0069 | .0065 | .0061 | .0058 | .0055 |

*For use with $\Sigma v$.

## 17. DATA WITH REGARD TO VARIOUS ELEMENTS

| | Symbol. | Valence. | Relative Atomic Weight Oxygen = 16. | Gms. per c.c. Liquid or Solid. | Specific Heat. 0° C +. | Coeff. of Linear Expansion. Mean 0° to 100° C. ($t$ = 40° C.) $\times 10^{-6}$. | Specific Resistance in Microhms per c.c. at 0° C. | % Increase of Resistance for 1° C. at 20° C. | Electrochemical Equivalent in gms. per Coulomb $\times 1000$. | Melting Point °C. | Boiling Point °C. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ‡276 | 66 | 227 | 219 | 271 | 270 | 276 | 207 | 208 |
| Aluminum..... | Al | 3 | 27.1 | 2.58 | 0.209 | .222 | 2.8 | .... | .0936 | 658 | 1800 |
| Antimony..... | Sb | 3 | 120.2 | 6.62 | .050 | .1056 | 40.6 | .389 | .4152 | 630 | 1440 |
| Antimony..... | Sb | 5 | 120.2 | .... | ..... | ..... | ..... | .... | .2491 | .... | .... |
| Argon..... | A | . | 39.9 | 1.38 | ..... | ..... | ..... | .... | .... | − 188 | − 186.1 |
| Barium..... | Ba | 2 | 137.37 | 3.75 | .068 | ..... | ..... | .... | .7118 | 850 | .... |
| Bismuth..... | Bi | 3 | 208.0 | 9.70 | .030 | .1316 | 108.7 | .354 | .7185 | 270 | 1430 |
| Bismuth..... | Bi | 5 | 208.0 | .... | ..... | ..... | ..... | .... | .4311 | .... | .... |
| Bromine..... | Br | 1 | 79.92 | 3.15 | .084 | ..... | ..... | .... | .8282 | − 7.3 | 61.1 |
| Cadmium..... | Cd | 2 | 112.4 | 8.37 | .055 | .3159 | ..... | .... | .5824 | 321 | 770 |
| Calcium..... | Ca | 2 | 40.09 | 1.52 | .157 | ..... | ..... | .... | .2077 | 805 | .... |
| Carbon..... | C | 4 | 12.0 | 3.50 | .160 | ..... | ..... | .... | .0313 | 3500 | 3600 |
| Chlorine..... | Cl | 1 | 35.46 | 1.51 | .226 | ..... | ..... | .... | .3675 | − 102 | − 33.6 |
| Chromium..... | Cr | 3 | 52.0 | 6.92 | .104 | ..... | ..... | .... | .1797 | 1505 | 2200 |
| Chromium..... | Cr | 6 | 52.0 | .... | ..... | ..... | ..... | .... | .090 | .... | .... |

## 17. DATA WITH REGARD TO VARIOUS ELEMENTS—*Continued*

| | Symbol. | Valence. | Relative Atomic Weight Oxygen = 16. | Gms. per c.c. Liquid or Solid. | Specific Heat. 0° C. + | Coeff. of Linear Expansion. Mean 0° to 100° C. (*= 40° C.) ×10⁻⁴. | Specific Resistance in Microhms per c.c. at 0° C. | % Increase of Resistance for 1° C. at 20° C. | Electrochemical Equivalent in gms. per Coulomb×1000. | Melting Point °C. | Boiling Point °C. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cobalt | Co | 2 | 58.97 | 8.71 | .103 | .1236* | 9.8 | .... | .3061 | 1490 | .... |
| Cobalt | Co | 3 | 58.97 | .... | ..... | ..... | ..... | .... | .2041 | .... | .... |
| Copper | Cu | 1 | 63.57 | 8.87 | .0942 | .1666 | 1.64 | .388 | .6588 | 1083 | 2310 |
| Copper | Cu | 2 | 63.57 | .... | ..... | ..... | ..... | .... | .3290 | .... | .... |
| Fluorine | F | 1 | 19.0 | 1.14 | ..... | ..... | ..... | .... | .1968 | − 223 | − 187 |
| Gold | Au | 3 | 197.2 | 19.3 | .0316 | .1470 | 2.07 | .365 | .6818 | 1063 | .... |
| Helium | He | − | 4.0 | .... | ..... | ..... | ..... | .... | .... | ..... | − 267 |
| Hydrogen | H | 1 | 1.008 | .07 | ..... | ..... | ..... | .... | .0104 | − 259 | − 252.6 |
| Iodine | I | 1 | 126.92 | 4.8 | .054 | ..... | ..... | .... | 1.3153 | 114 | 200 |
| Iridium | Ir | 4 | 193.1 | 22.4 | .0323 | ..... | ..... | .... | .5003 | 2250 | .... |
| Iron | Fe | 2 | 55.85 | 7.87 | .1152 | .1089 | 9.7–12. | .513 | .2894 | 1520 | 2450 |
| Iron | Fe | 3 | 55.85 | .... | ..... | ..... | ..... | .... | .1929 | .... | .... |
| Lead | Pb | 2 | 207.1 | 11.0 | .030 | .2709 | 18.4–19.6 | .387 | 1.0731 | 327 | 1525 |
| Lithium | Li | 1 | 7.0 | .534 | .795 | ..... | ..... | .... | .0725 | 186 | 1400 |
| Magnesium | Mg | 2 | 24.32 | 1.70 | .249 | .2694* | 4.1–5. | .... | .1260 | 651 | 1120 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Manganese..... | Mn | 2 | 54.93 | 7.40 | .121 | ..... | ..... | .... | .2846 | 1225 | 1900 |
| Manganese..... | Mn | 4 | 54.93 | .... | ..... | ..... | ..... | .... | .1423 | .... | .... |
| Mercury..... | Hg | 1 | 200.0 | 13.596 | .0335 | ..... | ..... | .072 | 2.0727 | − 39 | 357 |
| Mercury..... | Hg | 2 | 200.0 | .... | ..... | ..... | ..... | .... | 1.0363 | .... | .... |
| Neon..... | Ne | − | 20.0 | .... | ..... | ..... | ..... | .... | .... | − 252 | .... |
| Nickel..... | Ni | 2 | 58.68 | 8.80 | .109 | .1279* | 11.1 | .... | .3040 | 1450 | .... |
| Nickel..... | Ni | 3 | 58.68 | .... | ..... | ..... | ..... | .... | .2027 | .... | .... |
| Nitrogen..... | N | 3 | 14.01 | .81 | ..... | ..... | ..... | .... | .0484 | − 211 | − 195 |
| | | | ‡276 | 66 | 227 | 219 | 271 | 270 | 276 | 207 | 208 |
| Nitrogen..... | N | 5 | 14.01 | .... | ..... | ..... | ..... | .... | .0290 | .... | |
| Oxygen..... | O | 2 | 16.0 | 1.14 | ..... | ..... | ..... | .... | .0829 | − 230 | − 182.7 |
| Palladium..... | Pd | 2 | 106.7 | 11.4 | .059 | .1176* | 12.1 | .... | .5528 | 1545 | |
| Palladium..... | Pd | 5 | 106.7 | .... | ..... | ..... | ..... | .... | .2211 | .... | |
| Phosphorus..... | P | 3 | 31.0 | 2.34 | .20 | 1.253* | ..... | .... | .1071 | 44.2 | 288 |
| Phosphorus..... | P | 5 | 31.0 | .... | ..... | ..... | ..... | .... | .0642 | .... | |
| Platinum..... | Pt | 2 | 195.0 | 21.5 | .0323 | .0899* | 9.0–15.5 | .354 | 1.0104 | 1755 | |
| Platinum..... | Pt | 4 | 195.0 | .... | ..... | ..... | ..... | .... | .5052 | .... | |
| Potassium..... | K | 1 | 39.1 | .85 | .170 | ..... | 25.1 | .4052 | .... | 62.5 | 712 |
| Radium..... | Ra | 2 | 226.4 | .... | ..... | ..... | ..... | .... | .... | .... | .... |
| Selenium..... | Se | 2 | 79.2 | 4.5 | .068 | .6604 | ..... | .... | .4104 | ..... | 690 |
| Silicon..... | Si | 4 | 28.3 | 2.2 | .1838 | .0763* | ..... | .... | .0733 | 1420 | .... |
| Silver..... | Ag | 1 | 107.88 | 10.5 | .0559 | .1921* | 1.5–1.7 | .377 | 1.1180 | 961 | 1955 |
| Sodium..... | Na | 1 | 23.0 | .95 | .253 | ..... | ..... | .... | 0.2384 | 97 | 750 |

‡Number of Smithsonian Table.

## 17. DATA WITH REGARD TO VARIOUS ELEMENTS—*Continued*

| | Symbol. | Valence. | Relative Atomic Weight Oxygen = 16. | Gms. per c.c. Liquid or Solid. | Specific Heat. 0° C+. | Coeff. of linear Expansion. Mean 0° to 100° C. (*= 40 C.) ×10⁻⁴. | Specific Resistance in Microhms per c.c. at 0° C. | % Increase of Resistance for 1°C. at 20°C. | Electrochemical Equivalent in gms. per Coulomb × 1000. | Melting Point °C. | Boiling Point °C. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Strontium..... | Sr | 2 | 87.62 | 2.55 | ..... | ..... | ..... | .... | .4540 | .... | .... |
| Sulphur..... | S | 2 | 32.07 | 2.0 | .181 | 1.180 | ..... | .... | .1662 | 115 | 444.7 |
| Tantalum..... | Ta | 5 | 181.0 | 10.6 | .0333 | ..... | ..... | .... | .3751 | 2800 | .... |
| Tellurium..... | Te | 2 | 127.5 | 6.25 | .048 | .3687 | ..... | .... | .6606 | 451 | 1390 |
| Thallium..... | Tl | 1 | 204.0 | 11.8 | .0326 | .3021* | ..... | .... | 2.1141 | 301 | 1700 |
| Thorium..... | Th | 2 | 232.42 | 11.0 | .0276 | ..... | ..... | .... | 1.2043 | 1700 | .... |
| Tin..... | Sn | 2 | 119.0 | 7.18 | .055 | .2296 | 9.53 | .365 | .6166 | 231.9 | 2270 |
| Tungsten..... | W | 6 | 184.0 | 18.8 | .0336 | ..... | ..... | .... | .3178 | 2950 | .... |
| Uranium..... | U | 2 | 238.5 | 18.7 | .028 | ..... | ..... | .... | 1.2358 | 2000 | .... |
| Uranium..... | U | 3 | 238.5 | .... | ..... | ..... | ..... | .... | .8238 | .... | |
| Vanadium..... | V | 3 | 51.2 | 5.5 | .1153 | ..... | ..... | .... | .1768 | 1750 | .... |
| Zinc..... | Zn | 2 | 65.37 | 7.19 | .0935 | .2976 | 6.00 | .365 | .3385 | 419 | 930 |

‡ Number of Smithsonian Table.

## 18. ALLOYS

| | Density. | Specific Heat. | Coefficient of Thermal Expansion. | Specific Resistance. Microms per c.c. | Temp. Coefficient % |
|---|---|---|---|---|---|
| Brass (70 Cu. + 30 Zn).. | 8.44 | .0883 | $.1859 \times 10^{-4}$ | 8.0 | .40 |
| German silver..... | 8.3 | .0946 | .1836 | 31.5 | .025 |
| Platinum and Iridium.. | 21.62 | ... | .0884 | 29.90 | .087 |
| Constantine..... | 8.97 | ... | ..... | 48.30 | .0014 |
| Manganin..... | 8.6 | ... | ..... | 39.14 | .0017 |
| $I_a I_a$ (hard)..... | ... | ... | ..... | 50.2 | − .0011 |
| $I_a I_a$ (soft)..... | ... | ... | ..... | 47.1 | +0.0005 |

## 19. GASES

| Gas. | Specific Gravity. | Grams per Liter. | Pounds per Cu.ft. | Specific Heat at Constant Pressure. | | Exp. Coeff. 76. Constant Volume. $\times 10^{-}$ | Exp. Coeff. 76. Constant Pres. $\times 10^{-2}$ |
|---|---|---|---|---|---|---|---|
| | *71 | 71 | 71 | 232 | | 223 | |
| Air..... | 1.000 | 1.2928 | .08071 | (0–100°) | 0.2374 | .36650 | .3671 |
| Ammonia..... | 0.597 | 0.7621 | .04758 | (23–100) | 0.5202 | | |
| Carbon dioxide.... | 1.5291 | 1.9652 | .12269 | (15–100) | .2025 | .36856 | .3710 |
| Carbon monoxide.. | 0.9672 | 1.2506 | .07807 | (23–99) | 0.2425 | .36667 | .3669 |
| Chlorine..... | 2.491 | 3.1666 | .19769 | (13–202) | 0.1241 | | |
| Coal gas from... | 0.320 | 0.414 | .02583 | | | | |
| Coal gas to..... | 0.740 | 0.957 | .05973 | | | | |
| Hydrogen..... | 0.0696 | 0.09004 | .005621 | (21–100) | 3.410 | .36504 | .3660 |
| Nitrogen..... | 0.9673 | 1.2542 | .07829 | ( 0–200) | 0.2438 | .36682 | |
| Oxygen..... | 1.0530 | 1.4292 | .08922 | (13–207) | 0.2175 | .36681 | |
| Steam at 100°..... | 0.4690 | 0.5810 | .0363 | 100° | 0.421 | ..... | .4187 |

* No. of Smithsonian Table.

## 20. DENSITY AND VOLUME OF WATER AT DIFFERENT TEMPERATURES

Smithsonian Table No. 75.

| ° C. | Density. | Volume. | ° C. | Density. | Volume. | ° C. | Density. | Volume. | ° C. | Density. | Volume. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.99987 | 1.00013 | 20 | 0.99823 | 1.00177 | 40 | 0.99224 | 1.00782 | 80 | 0.97183 | 1.02899 |
| +1 | 993 | 007 | 21 | 802 | 198 | 41 | 186 | 821 | 85 | .96865 | 1.03237 |
| 2 | 997 | 003 | 22 | 780 | 221 | 42 | 147 | 861 | 90 | 534 | 590 |
| 3 | 999 | 001 | 23 | 756 | 244 | 43 | 107 | 901 | 95 | 192 | 959 |
| 4 | 1.0000 | 1.0000 | 24 | 732 | 268 | 44 | 066 | 943 | 100 | 0.95838 | 1.04343 |
| 5 | 0.99999 | 1.0001 | 25 | 0.99707 | 1.00294 | 45 | 0.99025 | 1.00985 | 110 | 0.9510 | 1.0515 |
| 6 | 997 | 003 | 26 | 681 | 320 | 46 | 0.98982 | 1.01028 | 20 | 0.9434 | 1.0601 |
| 7 | 993 | 007 | 27 | 654 | 347 | 47 | 940 | 072 | 130 | 0.9352 | 1.0693 |
| 8 | 988 | 012 | 28 | 626 | 375 | 48 | 896 | 116 | 140 | 0.9264 | 1.0794 |
| 9 | 981 | 019 | 29 | 597 | 405 | 49 | 852 | 162 | 150 | 0.9173 | 1.0902 |
| 10 | 0.99973 | 1.00027 | 30 | 0.99567 | 1.00435 | 50 | 0.98807 | 1.01207 | 160 | 0.9075 | 1.1019 |
| 11 | 963 | 037 | 31 | 537 | 466 | 51 | 762 | 254 | 170 | 0.8973 | 1.1145 |
| 12 | 952 | 048 | 32 | 505 | 497 | 52 | 715 | 301 | 180 | 0.8866 | 1.1279 |
| 13 | 940 | 060 | 33 | 473 | 530 | 53 | 669 | 349 | 190 | 0.8750 | 1.1429 |
| 14 | 927 | 073 | 34 | 440 | 563 | 54 | 621 | 398 | 200 | 0.8628 | 1.1590 |
| 15 | 0.99913 | 1.00087 | 35 | 0.99406 | 1.00598 | 55 | 0.98573 | 1.01448 | 210 | 0.850 | 1.177 |
| 16 | 897 | 103 | 36 | 371 | 633 | 60 | 324 | 705 | 220 | 0.837 | 1.195 |
| 17 | 880 | 120 | 37 | 336 | 669 | 65 | 059 | 979 | 230 | 0.823 | 1.215 |
| 18 | 862 | 138 | 38 | 299 | 706 | 70 | 0.97781 | 1.02270 | 240 | 0.809 | 1.236 |
| 19 | 843 | 157 | 39 | 262 | 743 | 75 | 489 | 576 | 250 | 0.794 | 1.259 |

## 21. BOILING POINT OF WATER UNDER DIFFERENT BAROMETRIC PRESSURES

| Bar. Pressure. | .0 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 72. | 98.50 | 98.54 | 98.57 | 98.61 | 98.65 | 98.69 | 98.73 | 98.77 | 98.80 | 98.84 |
| 73. | 98.88 | 98.92 | 98.96 | 98.99 | 99.03 | 99.07 | 99.11 | 99.14 | 99.18 | 99.22 |
| 74. | 99.26 | 99.30 | 99.33 | 99.37 | 99.41 | 99.44 | 99.48 | 99.52 | 99.56 | 99.59 |
| 75. | 99.63 | 99.67 | 99.71 | 99.74 | 99.78 | 99.82 | 99.85 | 99.89 | 99.93 | 99.96 |
| 76. | 100.00 | 100.04 | 100.07 | 100.11 | 100.15 | 100.18 | 100.22 | 100.26 | 100.29 | 100.33 |
| 77. | 100.36 | 100.40 | 100.44 | 100.47 | 100.51 | 100.55 | 100.58 | 100.62 | 100.65 | 100.69 |

## 22. BAROMETER CORRECTION AT DIFFERENT TEMPERATURES, THE BAROMETER SCALE BEING CORRECT AT 62° F. (BRASS SCALE)

**The correction is to be subtracted from the observed height.**

| Temp. F°. | Pressure in Inches. | | | Temp. F°. | Pressure in Inches. | | |
|---|---|---|---|---|---|---|---|
| | 28 | 29 | 30 | | 28 | 29 | 30 |
| 64 | −.090 | .093 | .096 | 74 | −.115 | .119 | .123 |
| 65 | .092 | .095 | .099 | 75 | .117 | .122 | .126 |
| 66 | .095 | .096 | .101 | 76 | .120 | .124 | .128 |
| 67 | .097 | 101 | .104 | 77 | .122 | .127 | .131 |
| 68 | .100 | .103 | .107 | 78 | .125 | .129 | .134 |
| 69 | .102 | .106 | .110 | 79 | .127 | .132 | .137 |
| 70 | .105 | .109 | .112 | 80 | .130 | .135 | .139 |
| 71 | .107 | .111 | .115 | 81 | .132 | .137 | .142 |
| 72 | .110 | .114 | .118 | 82 | .135 | .140 | .145 |
| 73 | .112 | .116 | .120 | 83 | .137 | .142 | .147 |

## 22a. BAROMETER CORRECTION. BRASS SCALE CORRECT AT 0°C, METRIC MEASURE

**The correction is to be subtracted from the observed height.**

| Height in mm. | $a$ in mm. for each °C. | Height in mm. | $a$ in mm. for each °C. | Height in mm. | $a$ in mm. for each °C. |
|---|---|---|---|---|---|
| 500 | 0.0813 | 600 | 0.0975 | 700 | 0.1137 |
| 510 | .0830 | 610 | .0992 | 710 | .1154 |
| 520 | .0846 | 620 | .1008 | 720 | .1170 |
| 530 | .0862 | 630 | .1024 | 730 | .1186 |
| 540 | .0878 | 640 | .1040 | 740 | .1202 |
| 550 | .0894 | 650 | .1056 | 750 | .1218 |
| 560 | .0911 | 660 | .1073 | 760 | .1235 |
| 570 | .0927 | 670 | .1089 | 770 | .1251 |
| 580 | .0943 | 680 | .1105 | 780 | .1267 |
| 590 | .0959 | 690 | .1121 | 790 | .1283 |
| | | | | 800 | .1290 |

## 23. PRESSURE OF SATURATED MERCURY VAPOR IN MM. OF MERCURY

| $t$° C. | Pressure. | $t$° C. | Pressure. | $t$°.C. | Pressure. | $t$°. C. | Pressure. | $t$°.C. | Pressure. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.00047 | 10 | 0.00080 | 20 | 0.00133 | 70 | 0.050 | 120 | 0.779 |
| 2 | .00052 | 12 | .00089 | 30 | .0029 | 80 | .093 | 130 | 1.240 |
| 4 | .00058 | 14 | .00099 | 40 | .0063 | 90 | .165 | 140 | 1.930 |
| 6 | .00064 | 16 | .00109 | 50 | .0130 | 100 | .285 | 150 | 2.930 |
| 8 | .00072 | 18 | .00121 | 60 | .0260 | 110 | .478 | 160 | 4.380 |

## 24. PRESSURE OF SATURATED AQUEOUS VAPOR IN MM. OF MERCURY

| $t°$ C. | Pressure. | $t°$ C. | Pressure. | $t°$ C. | Pressure. | $t°$ C. | Pressure. | $t°$ C. | Pressure. | $t°$ C. | Pressure. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.91 | 21 | 18.47 | 41 | 57.87 | 61 | 155.95 | 81 | 369.51 | 105 | 906.41 |
| 2 | 5.27 | 22 | 19.63 | 42 | 61.02 | 62 | 163.29 | 82 | 384.64 | 110 | 1075.37 |
| 3 | 5.66 | 23 | 20.86 | 43 | 64.31 | 63 | 170.92 | 83 | 400.29 | 115 | 1269.41 |
| 4 | 6.07 | 24 | 22.15 | 44 | 67.76 | 64 | 178.86 | 84 | 416.47 | 120 | 1491.28 |
| 5 | 6.51 | 25 | 23.52 | 45 | 71.36 | 65 | 187.10 | 85 | 433.19 | 125 | 1743.88 |
| 6 | 6.97 | 26 | 24.96 | 46 | 75.13 | 66 | 195.67 | 86 | 450.47 | 130 | 2030.28 |
| 7 | 7.47 | 27 | 26.47 | 47 | 79.07 | 67 | 204.56 | 87 | 468.32 | 135 | 2353.73 |
| 8 | 7.99 | 28 | 28.07 | 48 | 83.19 | 68 | 213.79 | 88 | 486.76 | 140 | 2717.63 |
| 9 | 8.55 | 29 | 29.74 | 49 | 87.49 | 69 | 223.37 | 89 | 505.81 | 145 | 3125.55 |
| 10 | 9.14 | 30 | 31.51 | 50 | 91.98 | 70 | 233.31 | 90 | 525.47 | 150 | 3581.23 |
| 11 | 9.77 | 31 | 33.37 | 51 | 96.66 | 71 | 243.62 | 91 | 545.77 | 155 | 4088.60 |
| 12 | 10.43 | 32 | 35.32 | 52 | 101.55 | 72 | 254.30 | 92 | 566.71 | 160 | 4651.60 |
| 13 | 11.14 | 33 | 37.37 | 53 | 106.65 | 73 | 265.38 | 93 | 588.33 | 165 | 5274.50 |
| 14 | 11.88 | 34 | 39.52 | 54 | 111.97 | 74 | 276.87 | 94 | 610.64 | 170 | 5961.70 |
| 15 | 12.67 | 35 | 41.78 | 55 | 117.52 | 75 | 288.76 | 95 | 633.66 | 175 | 6717.40 |
| 16 | 13.51 | 36 | 44.16 | 56 | 123.29 | 76 | 301.09 | 96 | 657.40 | 180 | 7546.40 |
| 17 | 14.40 | 37 | 46.65 | 57 | 129.31 | 77 | 313.85 | 97 | 687.88 | 185 | 8453.20 |
| 18 | 15.33 | 38 | 49.26 | 58 | 135.58 | 78 | 327.05 | 98 | 707.13 | 190 | 9442.70 |
| 19 | 16.32 | 39 | 52.00 | 59 | 142.10 | 79 | 340.73 | 99 | 733.16 | 195 | 10519.6 |
| 20 | 17.36 | 40 | 54.87 | 60 | 148.88 | 80 | 354.87 | 100 | 760.00 | 200 | 11689.0 |

## 25. DRY AND WET BULB HYGROMETER

This table gives the vapor pressure corresponding to various values of the differences $t-t_1$ between the readings of the dry and wet bulb thermometers and the temperature $t_1$ of the wet bulb thermometer. The table was calculated for a barometric pressure $B$ equal to 76 centimeters and a correction is given for each centimeter at the top of the columns. For pressures less than 76 the correction is to be added.

| $t_1$° C. | $t-t_1$ =0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | Difference per $t$° of $t-t_1$. |
|---|---|---|---|---|---|---|---|---|---|---|
| Correction for $B$ per cm. | | .013 | .026 | .040 | .053 | .066 | .079 | .092 | .106 | |
| | mm. | | | | | | Example: $t-t_1$ = 7.2 | | | |
| | | | | | | | $t_1$ = 10.0 | | | |
| | | | | | | | $B$ = 74.5 | | | |
| **0** | 4.60 | 3.60 | 2.59 | 1.59 | 0.59 | | Tabular No. = 6.12 − | | | **0.100** |
| 1 | 4.94 | 3.93 | 2.92 | 1.92 | 0.92 | | 6 × .101 = 5.51 | | | 0.100 |
| 2 | 5.30 | 4.29 | 3.29 | 2.28 | 1.28 | 0.27 | Correction for $B$ = 1.5 × .048 = .07 | | | 0.100 |
| 3 | 5.69 | 4.68 | 3.68 | 2.67 | 1.66 | 0.66 | Hence, $p$ = 5.58 | | | 0.101 |
| 4 | 6.10 | 5.09 | 4.09 | 3.08 | 2.07 | 1.06 | 0.05 | | | 0.101 |
| **5** | 6.53 | 5.52 | 4.51 | 3.50 | 2.49 | 1.48 | 0.48 | | | **0.101** |
| 6 | 7.00 | 5.99 | 4.98 | 3.97 | 2.96 | 1.95 | 0.94 | | | 0.101 |
| 7 | 7.49 | 6.48 | 5.47 | 4.45 | 3.44 | 2.43 | 1.42 | 0.41 | | 0.101 |
| 8 | 8.02 | 7.01 | 5.99 | 4.98 | 3.97 | 2.96 | 1.94 | 0.93 | | 0.101 |
| 9 | 8.57 | 7.56 | 6.54 | 5.53 | 4.51 | 3.50 | 2.49 | 1.48 | 0.46 | 0.101 |
| **10** | 9.17 | 8.16 | 7.14 | 6.12 | 5.11 | 4.09 | 3.08 | 2.07 | 1.06 | **0.101** |
| 11 | 9.79 | 8.77 | 7.76 | 6.74 | 5.73 | 4.71 | 3.69 | 2.68 | 1.66 | 0.102 |
| 12 | 10.46 | 9.44 | 8.43 | 7.41 | 6.39 | 5.37 | 4.36 | 3.34 | 2.32 | 0.102 |
| 13 | 11.16 | 10.14 | 9.12 | 8.10 | 7.09 | 6.07 | 5.05 | 4.03 | 3.01 | 0.102 |
| 14 | 11.91 | 10.89 | 9.87 | 8.85 | 7.83 | 6.81 | 5.79 | 4.77 | 3.71 | 0.102 |
| **15** | 12.70 | 11.68 | 10.66 | 9.64 | 8.62 | 7.60 | 6.58 | 5.56 | 4.54 | **0.102** |
| 16 | 13.54 | 12.52 | 11.50 | 10.47 | 9.45 | 8.43 | 7.41 | 6.39 | 5.37 | 0.102 |
| 17 | 14.42 | 13.40 | 12.37 | 11.35 | 10.33 | 9.31 | 8.28 | 7.26 | 6.24 | 0.102 |
| 18 | 15.36 | 14.34 | 13.31 | 12.29 | 11.26 | 10.24 | 9.21 | 8.19 | 7.17 | 0.102 |
| 19 | 16.35 | 15.33 | 14.30 | 13.27 | 12.25 | 11.22 | 10.20 | 9.17 | 8.15 | 0.102 |
| **20** | 17.39 | 16.37 | 15.34 | 14.31 | 13.28 | 12.26 | 11.23 | 10.21 | 9.18 | **0.103** |

For higher or lower values of $t_1$ see the Smithsonian Tables.

## 26. WAVE-LENGTHS IN ÅNGSTRÖM UNITS ($10^{-8}$ cm.)

Smithsonian Table No. 151.

| | | | | | |
|---|---|---|---|---|---|
| Barium | 5535.7 | Hydrogen | 4101.8 | Potassium | 7668.5 |
| Caesium | 4555.4 | Hydrogen | 4340.7 | Potassium | 7701.9 |
| Caesium | 4593.3 | Hydrogen | 4861.5 | Rubidium | 6298.7 |
| Calcium | 5589.0 | Hydrogen | 6563.0 | Sodium | 5890.2 |
| Cadmium | 4799.9 | Lithium | 6708.2 | Sodium | 5896.2 |
| Cadmium | 5085.8 | Magnesium | 5167.5 | Strontium | 4607.5 |
| Cadmium | 6438.5 | Magnesium | 5172.9 | Strontium | 5481.2 |
| Helium | 5875.8 | Magnesium | 5183.8 | Strontium | 6408.6 |
| Helium | 5876.2 | Mercury | 5461.0 | Thallium | 5350.6 |

## 27. INDEX OF REFRACTION

| | *D* Line. | | *D* Line. |
|---|---|---|---|
| Water | 1.3335 | Benzol | 1.5014 |
| Alcohol | 1.3624 | Ether, ethyl | 1.3560 |
| Carbon bisulphide | 1.6293 | Chloroform | 1.4490 |
| Glass, light crown ($\delta = 2.50$) | 1.5280 | Glycerine | 1.4729 |
| Glass, heavy crown ($\delta = 3.00$) | 1.5604 | Aluminum | 1.44 |
| Glass, light flint ($\delta = 2.87$) | 1.5410 | Copper | 0.641 |
| Glass, heavy flint ($\delta = 4.22$) | 1.7102 | Gold | 0.366 |
| Quartz, crystal +ord | 1.5442 | Iron | 2.36 |
| Quartz, crystal+ext | 1.5533 | Silver | 0.181 |
| Iceland spar, ord | 1.6585 | Antimony | 3.04 |
| Iceland spar, ext | 1.4864 | Air, 0° 76 cm | 1.000293 |

## 28. ELASTIC CONSTANTS

Dynes per sq.cm.

| | Young's Modulus. | Rigidity. | Elastic Limit. | Velocity of Sound. Cm. per Sec. |
|---|---|---|---|---|
| Aluminum | $6.5 \times 10^{11}$ | $2.4\text{–}3.3 \times 10^{11}$ | ..... | $5.10 \times 10^{5}$ |
| Brass | 7.7–9.3 | 3.1–3.6 | $4.5\text{–}11 \times 10^{8}$ | 3.56 |
| Copper | 8.5–11.6 | 3.2–4.5 | 3–7 | 3.74 |
| Iron, cast | 6.9–10.8 | 2.6–4.2 | 7 | 4.32 |
| Iron, wrought | 19.3–20.9 | 7.7–8.5 | 20 | 5.06 |
| Iron, steel | 20.1–21.6 | 8.0–8.8 | 33–40 | 5.22 |
| Platinum | 15 | 6.1–6.5 | 14–26 | 2.69 |
| Silver | 7 | 2.6 | 3–11 | 2.61 |
| Wood | 0.7–1.5 | 0.07–0.12 | 1.5–2.4 | 3.0 |

## 29. COMPRESSIBILITY

| | Compression Per Megabarye. | Elasticity of Volume. |
|---|---|---|
| Water | $5.0\times10^{-5}$ | $2.03\times10^{10}$ |
| Mercury | $3.9\times10^{-6}$ | $2.60\times10^{11}$ |
| Glass | $2.6\times10^{-6}$ | $3.90\times10^{11}$ |

## 30. FUNDAMENTAL ELECTRICAL UNITS

### Magnetic Units

*Unit magnetic pole* is a pole of such strength that it repels a like pole of equal strength at a distance of one centimeter, in vacuo, with a force of one dyne.

*Magnetic field of unit intensity* is a field that acts on a unit magnetic pole placed in it with a force of one dyne.

### Electrostatic Units

*Electrostatic unit of quantity* of electricity is such a quantity which acts on an equal quantity at a distance of one centimeter, in vacuo, with a force of one dyne.

*Electrostatic unit of current* exists when an electrostatic unit of quantity of electricity flows per second past any point.

*Electrostatic unit of potential* exists at any point when it requires an erg of work to carry to it an electrostatic unit of electricity from an infinite distance or from a point of zero potential.

*Difference of potential of one electrostatic unit* exists between two points when it requires an erg of work to carry an electrostatic unit of quantity of electricity from one point to the other.

## Electromagnetic Units

*Electromagnetic unit of current* is that current which flowing in the circumference of a circle of one centimeter radius, exerts for every centimeter length a force of one dyne on a unit magnetic pole placed at the center.

*Electromagnetic unit of quantity* is that quantity of electricity which passes any point in a second when one electromagnetic unit of current is flowing. [This has been found by experiment to be $3\times10^{10}$ times greater than the electrostatic unit of quantity.]

*Electromagnetic unit of difference of potential* is the difference of potential that exists between two points when it requires an erg of work to carry an electromagnetic unit of quantity of electricity from one point to the other. [The unit is therefore smaller than the electrostatic unit of difference of potential by the factor $3\times10^{10}$.]

*Electromagnetic unit of resistance* is a resistance through which one electromagnetic unit of difference of potential causes one electromagnetic unit of current to flow.

## Practical Units

*Ampere* is one-tenth of an electromagnetic unit of current.

*Coulomb* is that quantity of electricity which passes any point in one second when the current is one ampere.

*Volt* is equal to $10^8$ electromagnetic units of difference of potential. [300 volts therefore equal one electrostatic unit of difference of potential.]

*Ohm* is a resistance through which one volt causes one ampere to flow. [One ohm therefore equals $10^9$ electromagnetic units of resistance.]

*Joule* is equal to $10^7$ ergs of work. It is also equal to the work done on a coulomb of electricity in moving it through a difference of potential of one volt. Work in joules = $EIt$.

*Watt* is an electrical unit of power and represents work done at the rate of one joule per second. Power in watts = $EI$. A kilowatt = 1000 watts.

*Farad* is the practical unit of capacity. A body has a capacity of one farad when a coulomb of electricity produces a change of one volt in its potential. a *microfarad* = 0.000001 farad.

*Henry* is the practical unit of induction. The induction is one henry if an electromotive force of one volt is induced when the inducing current changes at the rate of one ampere per second. This applies to both self-induction and mutual induction. A *millihenry* = 0.001 henry.

*Maxwell* is the practical unit of magnetic flux. It represents a single line of force. The flux in a field is $n$ maxwells when it contains $n$ magnetic lines of force.

*Gauss* is the practical unit of magnetic intensity or of the density of magnetic flux. The magnetic intensity is one gauss when there is one maxwell (or one line of force) per square centimeter.

### 31. DEFINITIONS OF THE FUNDAMENTAL ELECTRICAL UNITS BY THE LONDON CONFERENCE (1908)

These fundamental units are:

1. The *Ohm*, the unit of electric resistance, which has the value of 1,000,000,000 ($10^9$) in terms of the centimeter and the second;

2. The *Ampere*, the unit of electric current, which has the value of one-tenth (0.1) in terms of the centimeter, gram, and second;

3. The *Volt*, the unit of electromotive force, which

has the value of 100,000,000 ($10^8$) in terms of the centimeter, gram, and second;

4. The *Watt*, the unit of power, which has the value 10,000,000 ($10^7$) in terms of the centimeter, the gram, and the second.

As a system of units representing the above and sufficiently near for the purpose of electrical measurements, and as a basis for legislation, the conference recommended the adoption of the international ohm, the international ampere, the international volt, and the international watt, defined as follows:

1. The *International Ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 centimeters.

2. The *International Ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.00111800 of a gram per second.

3. The *International Volt* is the electrical pressure which, when steadily applied to a conductor the resistance of which is one international ohm will produce a current of one international ampere.

4. The *International Watt* is the energy expended per second by an unvarying electric current of one international ampere under an electric pressure of one international volt.

## 32. THE ELECTROMOTIVE FORCE OF THE WESTON NORMAL CELL

The electromotive force of the Weston normal cell derived from the international ohm and the international ampere according to the resolutions of the London Conference is, within one part in 10,000,

$$E = 1.01830 \text{ international volts at } 20° \text{ C.}$$

The formula for the temperature coefficient of the Weston normal cell adopted by the London Conference, based on the investigations of the Bureau of Standards, is as follows:

$$E_t = E_{20} - 0.0000406\ (t-20°) - 0.00000095\ (t-20)^2 + 0.00000001\ (t-20)^3.$$

## 33. RELATION BETWEEN ELECTRICAL UNITS

1 amp. = 0.1 electromagnetic units = $3 \times 10^9$ electrostatic units.

| | | | |
|---|---|---|---|
| 1 volt | $= 10^8$ | " | $= \frac{1}{300}$ " |
| 1 ohm | $= 10^9$ | " | $= \frac{1}{9 \times 10^{11}}$ " |
| 1 farad | $= \frac{1}{10^9}$ | " | $= 9 \times 10^{11}$ " |
| 1 henry | $= 10^9$ | " | $= \frac{1}{9 \times 10^{11}}$ " |

## 34. SQUARES, CUBES AND RECIPROCALS

| Nos. | Squares. | Cubes. | Reciprocals. | Nos. | Squares. | Cubes. | Reciprocals. |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1.000000000 | 51 | 26 01 | 132 651 | .019607843 |
| 2 | 4 | 8 | .500000000 | 52 | 27 04 | 140 608 | .019230769 |
| 3 | 9 | 27 | .333333333 | 53 | 28 09 | 148 877 | .018867925 |
| 4 | 16 | 64 | .250000000 | 54 | 29 16 | 157 464 | .018518519 |
| 5 | 25 | 125 | .200000000 | 55 | 30 25 | 166 375 | .018181818 |
| 6 | 36 | 216 | .166666667 | 56 | 31 36 | 175 616 | .017857143 |
| 7 | 49 | 343 | .142857143 | 57 | 32 49 | 185 193 | .017543860 |
| 8 | 64 | 512 | .125000000 | 58 | 33 64 | 195 112 | .017241379 |
| 9 | 81 | 729 | .111111111 | 59 | 34 81 | 205 379 | .016949153 |
| 10 | 1 00 | 1 000 | .100000000 | 60 | 36 00 | 216 000 | .016666667 |
| 11 | 1 21 | 1 331 | .090909091 | 61 | 37 21 | 226 981 | .016393443 |
| 12 | 1 44 | 1 728 | .083333333 | 62 | 38 44 | 238 328 | .016129032 |
| 13 | 1 69 | 2 197 | .076923077 | 63 | 39 69 | 250 047 | .015873016 |
| 14 | 1 96 | 2 744 | .071428571 | 64 | 40 96 | 262 144 | .015625000 |
| 15 | 2 25 | 3 375 | .066666667 | 65 | 42 25 | 274 625 | .015384615 |
| 16 | 2 56 | 4 096 | .062500000 | 66 | 43 56 | 287 496 | .015151515 |
| 17 | 2 89 | 4 913 | .058823529 | 67 | 44 89 | 300 763 | .014925373 |
| 18 | 3 24 | 5 832 | .055555556 | 68 | 46 24 | 314 432 | .014705882 |
| 19 | 3 61 | 6 859 | .052631579 | 69 | 47 61 | 328 509 | .014492754 |
| 20 | 4 00 | 8 000 | .050000000 | 70 | 49 00 | 343 000 | .014285714 |
| 21 | 4 41 | 9 261 | .047619048 | 71 | 50 41 | 357 911 | .014084507 |
| 22 | 4 84 | 10 648 | .045454545 | 72 | 51 84 | 373 248 | .013888889 |
| 23 | 5 29 | 12 167 | .043478260 | 73 | 53 29 | 389 017 | .013698630 |
| 24 | 5 76 | 13 824 | .041666667 | 74 | 54 76 | 405 224 | .013513514 |
| 25 | 6 25 | 15 625 | .040000000 | 75 | 56 25 | 421 875 | .013333333 |
| 26 | 6 76 | 17 576 | .038461538 | 76 | 57 76 | 438 976 | .013157895 |
| 27 | 7 29 | 19 683 | .037037037 | 77 | 59 29 | 456 533 | .012987013 |
| 28 | 7 84 | 21 952 | .035714286 | 78 | 60 84 | 474 552 | .012820513 |
| 29 | 8 41 | 24 389 | .034482759 | 79 | 62 41 | 493 039 | .012658228 |
| 30 | 9 00 | 27 000 | .033333333 | 80 | 64 00 | 512 000 | .012500000 |
| 31 | 9 61 | 29 791 | .032258065 | 81 | 65 61 | 531 441 | .012345679 |
| 32 | 10 24 | 32 768 | .031250000 | 82 | 67 24 | 551 368 | .012195122 |
| 33 | 10 89 | 35 937 | .030303030 | 83 | 68 89 | 571 787 | .012048193 |
| 34 | 11 56 | 39 304 | .029411765 | 84 | 70 56 | 592 704 | .011904762 |
| 35 | 12 25 | 42 875 | .028571429 | 85 | 72 25 | 614 125 | .011764706 |
| 36 | 12 96 | 46 656 | .027777778 | 86 | 73 96 | 636 056 | .011627907 |
| 37 | 13 69 | 50 653 | .027027027 | 87 | 75 69 | 658 503 | .011494253 |
| 38 | 14 44 | 54 872 | .026315789 | 88 | 77 44 | 681 472 | .011363636 |
| 39 | 15 21 | 59 319 | .025641026 | 89 | 79 21 | 704 969 | .011235955 |
| 40 | 16 00 | 64 000 | .025000000 | 90 | 81 00 | 729 000 | .011111111 |
| 41 | 16 81 | 68 921 | .024390244 | 91 | 82 81 | 753 571 | .010989011 |
| 42 | 17 64 | 74 088 | .023809524 | 92 | 84 64 | 778 688 | .010869565 |
| 43 | 18 49 | 79 507 | .023255814 | 93 | 86 49 | 804 357 | .010752688 |
| 44 | 19 36 | 85 184 | .022727273 | 94 | 88 36 | 830 584 | .010638298 |
| 45 | 20 25 | 91 125 | .022222222 | 95 | 90 25 | 857 375 | .010526316 |
| 46 | 21 16 | 97 336 | .021739130 | 96 | 92 16 | 884 736 | .010416667 |
| 47 | 22 09 | 103 823 | .021276600 | 97 | 94 09 | 912 673 | .010309278 |
| 48 | 23 04 | 110 592 | .020833333 | 98 | 96 04 | 941 192 | .010204082 |
| 49 | 24 01 | 117 649 | .020408163 | 99 | 98 01 | 970 299 | .010101010 |
| 50 | 25 00 | 125 000 | .020000000 | 100 | 100 00 | 1 000 000 | .010000000 |

## 35. TRIGONOMETRIC FUNCTIONS

| Arc. | Angle. | Sin. | Tan. | Sec. | Cosec. | Cot. | Cos. | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0 | 0.0000 | 0.0000 | 1.0000 | ∞ | ∞ | 1.0000 | 90 | 1.5708 |
| 0.0175 | 1 | 0.0175 | 0.0175 | 1.0002 | 57.2987 | 57.2900 | 0.9998 | 89 | 1.5533 |
| 0.0349 | 2 | 0.0349 | 0.0349 | 1.0006 | 28.6537 | 28.6363 | 0.9994 | 88 | 1.5359 |
| 0.0524 | 3 | 0.0523 | 0.0524 | 1.0014 | 19.1073 | 19.0811 | 0.9986 | 87 | 1.5184 |
| 0.0698 | 4 | 0.0698 | 0.0699 | 1.0024 | 14.3356 | 14.3007 | 0.9976 | 86 | 1.5010 |
| 0.0873 | 5 | 0.0872 | 0.0875 | 1.0038 | 11.4737 | 11.4301 | 0.9962 | 85 | 1.4835 |
| 0.1047 | 6 | 0.1045 | 0.1051 | 1.0055 | 9.5668 | 9.5144 | 0.9945 | 84 | 1.4661 |
| 0.1222 | 7 | 0.1219 | 0.1228 | 1.0075 | 8.2055 | 8.1443 | 0.9925 | 83 | 1.4486 |
| 0.1396 | 8 | 0.1392 | 0.1405 | 1.0098 | 7.1853 | 7.1154 | 0.9903 | 82 | 1.4312 |
| 0.1571 | 9 | 0.1564 | 0.1584 | 1.0125 | 6.3925 | 6.3138 | 0.9877 | 81 | 1.4137 |
| 0.1745 | 10 | 0.1763 | 0.1763 | 1.0154 | 5.7588 | 5.6713 | 0.9848 | 80 | 1.3963 |
| 0.1920 | 11 | 0.1908 | 0.1944 | 1.0187 | 5.2408 | 5.1446 | 0.9816 | 79 | 1.3788 |
| 0.2094 | 12 | 0.2079 | 0.2126 | 1.0223 | 4.8097 | 4.7046 | 0.9781 | 78 | 1.3614 |
| 0.2269 | 13 | 0.2250 | 0.2309 | 1.0263 | 4.4454 | 4.3315 | 0.9744 | 77 | 1.3439 |
| 0.2443 | 14 | 0.2419 | 0.2493 | 1.0306 | 4.1336 | 4.0108 | 0.9703 | 76 | 1.3264 |
| 0.2618 | 15 | 0.2588 | 0.2679 | 1.0353 | 3.8637 | 3.7321 | 0.9659 | 75 | 1.3090 |
| 0.2793 | 16 | 0.2756 | 0.2867 | 1.0403 | 3.6280 | 3.4874 | 0.9613 | 74 | 1.2915 |
| 0.2967 | 17 | 0.2924 | 0.3057 | 1.0457 | 3.4203 | 3.2709 | 0.9563 | 73 | 1.2741 |
| 0.3142 | 18 | 0.3090 | 0.3249 | 1.0515 | 3.2361 | 3.0777 | 0.9511 | 72 | 1.2566 |
| 0.3316 | 19 | 0.3256 | 0.3443 | 1.0576 | 3.0716 | 2.9042 | 0.9455 | 71 | 1.2392 |
| 0.3491 | 20 | 0.3420 | 0.3640 | 1.0642 | 2.9238 | 2.7475 | 0.9397 | 70 | 1.2217 |
| 0.3665 | 21 | 0.3584 | 0.3839 | 1.0711 | 2.7904 | 2.6051 | 0.9336 | 69 | 1.2043 |
| 0.3840 | 22 | 0.3746 | 0.4040 | 1.0785 | 2.6695 | 2.4751 | 0.9272 | 68 | 1.1868 |
| 0.4014 | 23 | 0.3907 | 0.4245 | 1.0864 | 2.5593 | 2.3559 | 0.9205 | 67 | 1.1694 |
| 0.4189 | 24 | 0.4067 | 0.4452 | 1.0946 | 2.4586 | 2.2460 | 0.9135 | 66 | 1.1519 |
| 0.4363 | 25 | 0.4226 | 0.4663 | 1.1034 | 2.3662 | 2.1445 | 0.9063 | 65 | 1.1345 |
| 0.4538 | 26 | 0.4384 | 0.4877 | 1.1126 | 2.2812 | 2.0503 | 0.8988 | 64 | 1.1170 |
| 0.4712 | 27 | 0.4540 | 0.5095 | 1.1223 | 2.2027 | 1.9626 | 0.8910 | 63 | 1.0996 |
| 0.4887 | 28 | 0.4695 | 0.5317 | 1.1326 | 2.1301 | 1.8807 | 0.8829 | 62 | 1.0821 |
| 0.5061 | 29 | 0.4848 | 0.5543 | 1.1434 | 2.0627 | 1.8040 | 0.8746 | 61 | 1.0647 |
| 0.5236 | 30 | 0.5000 | 0.5774 | 1.1547 | 2.0000 | 1.7321 | 0.8660 | 60 | 1.0472 |
| 0.5411 | 31 | 0.5150 | 0.6009 | 1.1666 | 1.9416 | 1.6643 | 0.8572 | 59 | 1.0297 |
| 0.5585 | 32 | 0.5299 | 0.6249 | 1.1792 | 1.8871 | 1.6003 | 0.8480 | 58 | 1.0123 |
| 0.5760 | 33 | 0.5446 | 0.6494 | 1.1924 | 1.8361 | 1.5399 | 0.8387 | 57 | 0.9948 |
| 0.5934 | 34 | 0.5592 | 0.6745 | 1.2062 | 1.7883 | 1.4826 | 0.8290 | 56 | 0.9774 |
| 0.6109 | 35 | 0.5736 | 0.7002 | 1.2208 | 1.7434 | 1.4281 | 0.8192 | 55 | 0.9599 |
| 0.6283 | 36 | 0.5878 | 0.7265 | 1.2361 | 1.7013 | 1.3764 | 0.8090 | 54 | 0.9425 |
| 0.6458 | 37 | 0.6018 | 0.7536 | 1.2521 | 1.6616 | 1.3270 | 0.7986 | 53 | 0.9250 |
| 0.6632 | 38 | 0.6157 | 0.7813 | 1.2690 | 1.6243 | 1.2799 | 0.7880 | 52 | 0.9076 |
| 0.6807 | 39 | 0.6293 | 0.8098 | 1.2868 | 1.5890 | 1.2349 | 0.7771 | 51 | 0.8901 |
| 0.6981 | 40 | 0.6428 | 0.8391 | 1.3054 | 1.5557 | 1.1918 | 0.7660 | 50 | 0.8727 |
| 0.7156 | 41 | 0.6561 | 0.8693 | 1.3250 | 1.5243 | 1.1504 | 0.7547 | 49 | 0.8552 |
| 0.7330 | 42 | 0.6691 | 0.9004 | 1.3456 | 1.4945 | 1.1106 | 0.7431 | 48 | 0.8378 |
| 0.7505 | 43 | 0.6820 | 0.9325 | 1.3673 | 1.4663 | 1.0724 | 0.7314 | 47 | 0.8203 |
| 0.7679 | 44 | 0.6947 | 0.9657 | 1.3902 | 1.4396 | 1.0355 | 0.7193 | 46 | 0.8029 |
| 0.7854 | 45 | 0.7071 | 1.0000 | 1.4142 | 1.4142 | 1.0000 | 0.7071 | 45 | 0.7854 |
| | | Cos. | Cot. | Cosec. | Sec. | Tan. | Sin. | Angle. | Arc. |

## 36. LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

| Angle. | Log Sin. | Log Tan. | Log Sec. | Log Csc. | Log Cot. | Log Cos. | |
|---|---|---|---|---|---|---|---|
| 0 | $-\infty$ | $-\infty$ | 0.0000 | $\infty$ | $\infty$ | 0.0000 | 90 |
| 1 | 8.2419 | 8.2419 | 0.0001 | 1.7581 | 1.7581 | 9.9999 | 89 |
| 2 | 8.5428 | 8.5431 | 0.0003 | 1.4572 | 1.4569 | 9.9997 | 88 |
| 3 | 8.7188 | 8.7194 | 0.0006 | 1.2812 | 1.2806 | 9.9994 | 87 |
| 4 | 8.8436 | 8.8446 | 0.0011 | 1.1564 | 1.1554 | 9.9989 | 86 |
| 5 | 8.9403 | 8.9420 | 0.0017 | 1.0597 | 1.0580 | 9.9983 | 85 |
| 6 | 9.0192 | 9.0216 | 0.0024 | 0.9808 | 0.9784 | 9.9976 | 84 |
| 7 | 9.0859 | 9.0891 | 0.0032 | 0.9141 | 0.9109 | 9.9968 | 83 |
| 8 | 9.1436 | 9.1478 | 0.0042 | 0.8564 | 0.8522 | 9.9958 | 82 |
| 9 | 9.1943 | 9.1997 | 0.0054 | 0.8057 | 0.8003 | 9.9946 | 81 |
| 10 | 9.2397 | 9.2463 | 0.0066 | 0.7603 | 0.7537 | 9.9934 | 80 |
| 11 | 9.2806 | 9.2887 | 0.0081 | 0.7194 | 0.7113 | 9.9919 | 79 |
| 12 | 9.3179 | 9.3275 | 0.0096 | 0.6821 | 0.6725 | 9.9904 | 78 |
| 13 | 9.3521 | 9.3634 | 0.0113 | 0.6479 | 0.6366 | 9.9887 | 77 |
| 14 | 9.3837 | 9.3968 | 0.0131 | 0.6163 | 0.6032 | 9.9869 | 76 |
| 15 | 9.4130 | 9.4281 | 0.0151 | 0.5870 | 0.5719 | 9.9849 | 75 |
| 16 | 9.4403 | 9.4575 | 0.0172 | 0.5597 | 0.5425 | 9.9828 | 74 |
| 17 | 9.4659 | 9.4853 | 0.0194 | 0.5341 | 0.5147 | 9.9806 | 73 |
| 18 | 9.4900 | 9.5118 | 0.0218 | 0.5100 | 0.4882 | 9.9782 | 72 |
| 19 | 9.5126 | 9.5370 | 0.0243 | 0.4874 | 0.4630 | 9.9757 | 71 |
| 20 | 9.5341 | 9.5611 | 0.0270 | 0.4659 | 0.4389 | 9.9730 | 70 |
| 21 | 9.5543 | 9.5842 | 0.0298 | 0.4457 | 0.4158 | 9.9702 | 69 |
| 22 | 9.5736 | 9.6064 | 0.0328 | 0.4264 | 0.3936 | 9.9672 | 68 |
| 23 | 9.5919 | 9.6279 | 0.0360 | 0.4081 | 0.3721 | 9.9640 | 67 |
| 24 | 9.6093 | 9.6486 | 0.0393 | 0.3907 | 0.3514 | 9.9607 | 66 |
| 25 | 9.6259 | 9.6687 | 0.0427 | 0.3741 | 0.3313 | 9.9573 | 65 |
| 26 | 9.6418 | 9.6882 | 0.0463 | 0.3582 | 0.3118 | 9.9537 | 64 |
| 27 | 9.6570 | 9.7072 | 0.0501 | 0.3430 | 0.2928 | 9.9499 | 63 |
| 28 | 9.6716 | 9.7257 | 0.0541 | 0.3284 | 0.2743 | 9.9459 | 62 |
| 29 | 9.6856 | 9.7438 | 0.0582 | 0.3144 | 0.2562 | 9.9418 | 61 |
| 30 | 9.6990 | 9.7614 | 0.0625 | 0.3010 | 0.2386 | 9.9375 | 60 |
| 31 | 9.7118 | 9.7788 | 0.0669 | 0.2882 | 0.2212 | 9.9331 | 59 |
| 32 | 9.7242 | 9.7958 | 0.0716 | 0.2758 | 0.2042 | 9.9284 | 58 |
| 33 | 9.7361 | 9.8125 | 0.0764 | 0.2639 | 0.1875 | 9.9236 | 57 |
| 34 | 9.7476 | 9.8290 | 0.0814 | 0.2524 | 0.1710 | 9.9186 | 56 |
| 35 | 9.7586 | 9.8452 | 0.0866 | 0.2414 | 0.1548 | 9.9134 | 55 |
| 36 | 9.7692 | 9.8613 | 0.0920 | 0.2308 | 0.1387 | 9.9080 | 54 |
| 37 | 9.7795 | 9.8771 | 0.0977 | 0.2205 | 0.1229 | 9.9023 | 53 |
| 38 | 9.7893 | 9.8928 | 0.1035 | 0.2107 | 0.1072 | 9.8965 | 52 |
| 39 | 9.7989 | 9.9084 | 0.1095 | 0.2011 | 0.0916 | 9.8905 | 51 |
| 40 | 9.8081 | 9.9238 | 0.1157 | 0.1919 | 0.0762 | 9.8843 | 50 |
| 41 | 9.8169 | 9.9392 | 0.1222 | 0.1831 | 0.0608 | 9.8778 | 49 |
| 42 | 9.8255 | 9.9544 | 0.1289 | 0.1745 | 0.0456 | 9.8711 | 48 |
| 43 | 9.8338 | 9.9697 | 0.1359 | 0.1662 | 0.0303 | 9.8641 | 47 |
| 44 | 9.8418 | 9.9848 | 0.1431 | 0.1582 | 0.0152 | 9.8569 | 46 |
| 45 | 9.8495 | 0.0000 | 0.1505 | 0.1505 | 0.0000 | 9.8495 | 45 |
| | Log Cos. | Log Cot. | Log Csc. | Log Sec. | Log Tan. | Log Sin. | Angle. |

## 37. LOGARITHMS OF NUMBERS FROM 1 TO 1000

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 2 3 | 4 5 6 | 7 8 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 8 12 | 17 21 25 | 29 33 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 8 11 | 15 19 23 | 26 30 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 7 10 | 14 17 21 | 24 28 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 6 10 | 13 16 19 | 23 26 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 6 9 | 12 15 18 | 21 24 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 6 8 | 11 14 17 | 20 22 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 5 8 | 11 13 16 | 18 21 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 5 7 | 10 12 15 | 17 20 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 5 7 | 9 12 14 | 16 19 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 4 7 | 9 11 13 | 16 18 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 4 6 | 8 11 13 | 15 17 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 4 6 | 8 10 12 | 14 16 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 4 6 | 8 10 12 | 14 15 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 4 6 | 7 9 11 | 13 15 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 4 5 | 7 9 11 | 12 14 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 3 5 | 7 9 10 | 12 14 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 3 5 | 7 8 10 | 11 13 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 3 5 | 6 8 9 | 11 13 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 3 5 | 6 8 9 | 11 12 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 3 4 | 6 7 9 | 10 12 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 3 4 | 6 7 9 | 10 11 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 3 4 | 6 7 8 | 10 11 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 3 4 | 5 7 8 | 9 11 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 3 4 | 5 6 8 | 9 10 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 3 4 | 5 6 8 | 9 10 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 2 4 | 5 6 7 | 9 10 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 2 4 | 5 6 7 | 8 10 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 2 3 | 5 6 7 | 8 9 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 2 3 | 5 6 7 | 8 9 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 2 3 | 4 5 7 | 8 9 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 2 3 | 4 5 6 | 8 9 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 2 3 | 4 5 6 | 7 8 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 2 3 | 4 5 6 | 7 8 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 2 3 | 4 5 6 | 7 8 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 2 3 | 4 5 6 | 7 8 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 2 3 | 4 5 6 | 7 8 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 2 3 | 4 5 6 | 7 7 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 2 3 | 4 5 5 | 6 7 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 2 3 | 4 4 5 | 6 7 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 2 3 | 4 4 5 | 6 7 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 2 3 | 3 4 5 | 6 7 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 2 3 | 3 4 5 | 6 7 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 2 2 | 3 4 5 | 6 7 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 2 2 | 3 4 5 | 6 6 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 2 2 | 3 4 5 | 6 6 7 |

## LOGARITHMS OF NUMBERS FROM 1 TO 1000 (*Con.*)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 2 3 | 4 5 6 | 7 8 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 2 2 | 3 4 5 | 5 6 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 2 2 | 3 4 5 | 5 6 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 2 2 | 3 4 5 | 5 6 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 1 2 | 3 4 4 | 5 6 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 1 2 | 3 4 4 | 5 6 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 1 2 | 3 4 4 | 5 6 6 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 1 2 | 3 4 4 | 5 6 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 1 2 | 3 3 4 | 5 6 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 1 2 | 3 3 4 | 5 5 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 1 2 | 3 3 4 | 5 5 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 1 2 | 3 3 4 | 5 5 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 1 2 | 3 3 4 | 5 5 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 1 2 | 3 3 4 | 5 5 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 1 2 | 3 3 4 | 4 5 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 1 2 | 2 3 4 | 4 5 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 1 2 | 2 3 4 | 4 5 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 1 2 | 2 3 4 | 4 5 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 1 2 | 2 3 4 | 4 5 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 1 2 | 2 3 4 | 4 5 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1 1 2 | 2 3 4 | 4 5 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 1 2 | 2 3 3 | 4 5 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 1 2 | 2 3 3 | 4 5 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 1 2 | 2 3 3 | 4 4 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 1 2 | 2 3 3 | 4 4 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 1 2 | 2 3 3 | 4 4 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 1 2 | 2 3 3 | 4 4 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 1 2 | 2 3 3 | 4 4 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 1 2 | 2 3 3 | 4 4 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 1 2 | 2 3 3 | 4 4 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 1 2 | 2 3 3 | 4 4 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 1 2 | 2 3 3 | 4 4 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 1 2 | 2 3 3 | 4 4 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 1 1 | 2 2 3 | 3 4 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 1 1 | 2 2 3 | 3 4 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 1 1 | 2 2 3 | 3 4 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 1 1 | 2 2 3 | 3 4 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 1 1 | 2 2 3 | 3 4 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 1 1 | 2 2 3 | 3 4 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 1 1 | 2 2 3 | 3 4 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 1 1 | 2 2 3 | 3 4 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 1 1 | 2 2 3 | 3 4 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 1 1 | 2 2 3 | 3 4 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 1 1 | 2 2 3 | 3 4 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 1 1 | 2 2 3 | 3 4 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 1 1 | 2 2 3 | 3 3 4 |

## 38. APPROXIMATE EXPRESSIONS

when $a, b, \ldots$ and $a_1, b_1, \ldots$ are small.

$$(1+a)^2=1+2a. \qquad (1-a)^2=1-2a.$$

$$\sqrt{1+a}=1+\tfrac{1}{2}a. \qquad \sqrt{1-a}=1-\tfrac{1}{2}a.$$

$$\frac{1}{1+a}=1-a. \qquad \frac{1}{1-a}=1+a.$$

$$\frac{1}{(1+a)^2}=1-2a. \qquad \frac{1}{(1-a)^2}=1+2a.$$

$$\frac{1}{\sqrt{1+a}}=1-\tfrac{1}{2}a. \qquad \frac{1}{\sqrt{1-a}}=1+\tfrac{1}{2}a.$$

$$(1\pm a)(1\pm b)(1\pm c)\ \ldots =1\pm a\pm b\pm c\ldots.$$

$$\frac{(1\pm a)(1\pm b)\ldots}{(1\pm a_1)(1\pm b_1)\ldots} \qquad =1\pm a\pm b\ldots\mp a_1\mp b_1\ \ldots$$

$$\sqrt{ab}=\frac{a+b}{2}.$$

$$\sin a=a.$$

$$\cos a=1.$$

$$\tan a=a.$$

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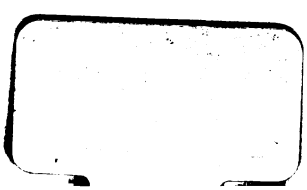
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